Optimal bidding strategies for electricity generation assets in the Day-Ahead, Intraday, and Balancing Markets

Miguel Ferreira
ISEL/Polytechnic University of Lisbon
Lisbon, Portugal
A47170@alunos.isel.pt

Jorge Sousa, João Lagarto
ISEL/Polytechnic University of Lisbon
INESC-ID
Lisbon, Portugal
jorge.sousa@isel.pt; joao.lagarto@isel.pt

***Abstract*— This study analyses optimal bidding strategies to maximize the profit of power generation plants in various markets, with a focus on the photovoltaic (PV) power plant. Although other technologies, such as thermal and reversible hydroelectric plants, could have been analyzed, PV stands out due to its rapid growth and contribution to sustainability in the energy transition process. The research uses an optimization model, to support the decision process of the participation of the PV in several electricity markets, namely day-ahead, intraday, secondary and tertiary reserves markets, under price-maker and price-taker assumptions. Simulations were carried out for a 24-hour period using real data from the MIBEL with the price-maker scenario being the most profitable, presenting higher revenues compared to the price-taker.**

***Index Terms*-- Day-ahead market, Intraday market, Profit maximization, Secondary reserve market, Tertiary reserve market**

I. Introduction

The growing complexity and dynamism of electricity markets demand a deep understanding of optimization strategies for different generation technologies. Solar photovoltaic (PV), in particular, has experienced significant growth due to its potential to contribute to sustainability and reduce reliance on fossil fuels. However, its non-dispatchable nature presents specific challenges in maximizing profits and integrating effectively into electricity markets [1].

Electricity markets consist of different segments that organize the trading of electricity in different time horizons [1]. The day-ahead market facilitates the trading of electricity for delivery in the following day, adjusting to day-ahead supply and demand conditions [2]. The intraday market offers opportunities for adjustments during the same day, responding to unforeseen variations [2]. The secondary reserve market and tertiary reserve market ensure the availability of additional capacity to accommodate fluctuations and guarantee the stability of the electricity grid [3], [4], [5], [6].

In this study, a model is presented based on [7], to analyze the behavior of a generating company owning a 100 MW PV power plant in several electricity markets, considering both price maker and price taker assumptions. The results obtained from the model are analyzed for each scenario, highlighting the main conclusions.

The paper is structured into three main sections. Section II presents the basis of the model, covering the restrictions of the residual demand curve, complementary equations for the different markets and the restrictions of the generating units. Section III analyses the results in the price-maker and price-taker scenarios. Finally, section IV presents the conclusions.

II. Model Description

The model used in this work extends the one presented in [7] to include the different sessions of the intraday market and the tertiary reserve market besides day-ahead and secondary reserve market. The model optimizes the decision-making process of a generating company that acts in the above-mentioned markets by maximizing its operational profit for a given period of time, for both price-taker and price-maker scenarios. The model was developed and implemented in Generic Algebraic Modeling System (GAMS) as a mixed integer linear programming problem. The mathematical formulation of the model, including its objective function, as well as its markets and generating units related constraints is presented in the appendix.

III. Results

In this section, results for a 24-hour period are presented considering a 100 MW PV power plant in both price-maker and price-taker scenarios. This type of plant has a very important characteristic: it produces only during a specific solar period, which in this case was considered to be from 9 AM to 9 PM. Data for the day-ahead and intraday markets were obtained from MIBEL's market operator OMIE [8] and data for the secondary and tertiary reserve markets were obtained from the

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Portuguese transmission system operator [9]. The simulations took a maximum of 4.3 seconds on a computer with a processor AMD Ryzen 5 4500 U with Radeon Graphics 2.38 GHz and 16 GB of RAM memory.

A. Price-Maker

In a price-maker framework, the generating company acknowledges that its decisions have a direct impact on market prices. This relationship can be analytically represented through the company's residual demand curve (see Appendix), which illustrates how market prices respond to variations in the quantity supplied by the company. Such a representation enables the company to strategically adjust its output in order to maximize operational profit.

Fig. 1 shows the final prices for each market in each hour, allowing the identification of the most profitable markets to trade.

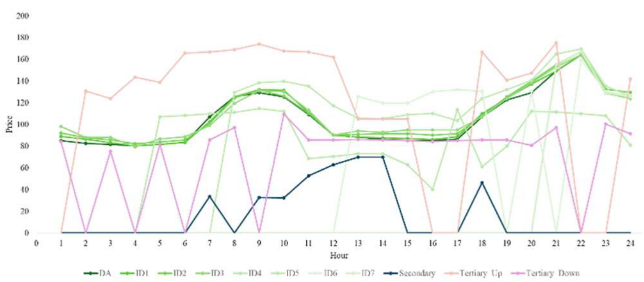

Figure 1. Final prices in each hour. Price in the secondary reserve market, in €/MW, prices in the DA, ID and tertiary markets, in €/MWh

The upward tertiary reserve market generally has the highest price. The intraday markets, 5 and 6, have the highest prices within their scheduling horizon. In the secondary reserve capacity market, there is significant variation, with high prices at the beginning of the day and low prices at the end.

Fig. 2 represents the energy sold in each market in each hour of the simulation period, taking into account the PV production.

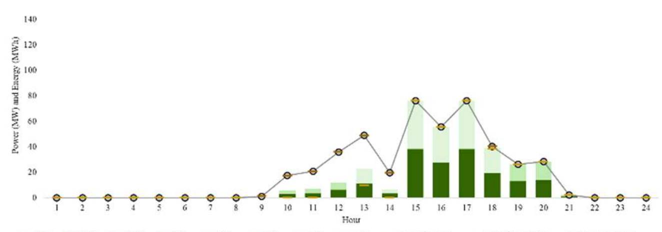

Figure 2. Energy traded by the PV power plant in the DA, ID, and tertiary markets and power traded in the secondary reserve market in each hour

In the simulations, it was considered that in each hour, from the total energy sold in the day-ahead market and in the different sessions of the intraday market, 50% is sold in the day-ahead market.

To better analyze the results presented in Fig. 2, two different groups of hours were considered. One group includes hours where the energy balance equals the total energy sold in the day-ahead and in the intraday markets, and the other group includes the hours where the energy balance is higher than the energy sold in these markets.

- Hours where energy balance equals the total energy sold in the day-ahead and in the intraday markets

  Between the hours 15 and 21, the generating company adopts the same energy selling strategy. In addition to the day-ahead market, the agent sells in the intraday market 6 between hour 15 and hour 18 and in the intraday 5 market between hours 19 and 21, where prices are higher. Between hours 15 and 17, and between hours 19 and 21, the generating company does not sell in the tertiary reserve market due to the high prices of downward energy, since the downward energy represents a cost. Despite the high prices of downward energy at hour 18, the generating company sells upward energy since the upward tertiary reserve price is quite high. At hour 18, to take advantage of the high price, the generating company sells a small amount of capacity in the secondary reserve market.

- Hours where the energy balance is higher than the energy sold in the day-ahead and in the intraday markets

  Between hours 9 and 14, the generating company sells energy in the day-ahead market and capacity in the secondary reserve market. Between hours 9 and 11, the generating company sells energy in the intraday 5 market, and between hour 13 and 14, in the intraday 6 market, taking advantage of the higher prices. During all these hours, it also sells upward energy in the tertiary reserve market due to higher prices compared to the intraday markets and sells capacity in the secondary reserve market.

Based on the previous figures, it is possible to determine the generating company's revenue in each market. Fig. 3 presents the cumulative revenues from the various markets during the simulation time, with the total hourly revenue depicted by the blue line.

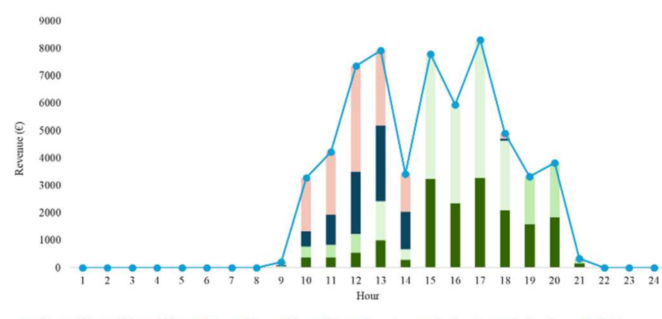

Figure 3. Revenue per market per hour

Although PV production at hour 12 is lower than at hour 18, the revenue is significantly higher. The analysis reveals that the amount of energy, the market price, and the hourly transactions influence the revenue. At hour 12, despite the reduced production, the generating company sells energy in the day-ahead and intraday markets at low prices. To maximize profit, the agent directs more selling offers to markets with higher prices, such as tertiary reserve and secondary reserve capacity markets. This contrasts with hour 18, where the agent is limited to the day-ahead and intraday markets. The total revenue for this day, for the PV plant under study, is 60883 €.

B. Price-Taker

In this scenario, the production of the PV plant in the various markets is optimized without considering the influence of the generating company on the final price. The prices before and after the actions of the generating company are the same, and the bidding strategy is built based on this assumption.

For the price-taker scenario, the prices of the different markets are depicted in Fig. 4. Analyzing these prices allows us to identify the most advantageous markets to sell energy in each hour and to maximize profit.

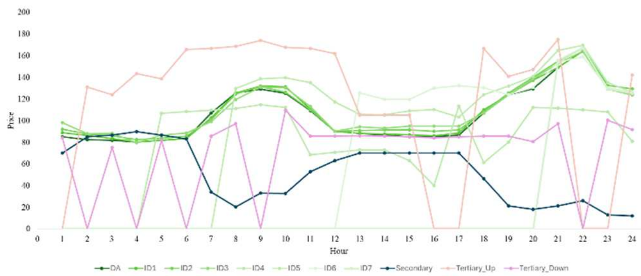

Figure 4. Initial prices in each hour. Price in the secondary reserve market, in €/MW, prices in the DA, ID amd tertiary markets, in €/MWh

Generally, the most profitable market is the upward tertiary reserve market since it presents the highest prices. The most profitable intraday markets are intraday 5 and 6 within their scheduling horizon. Secondary reserve market prices vary significantly, with high values at the beginning of the day and low values at the end.

Fig. 5 presents the energy sold per market per hour, taking into account the PV production period.

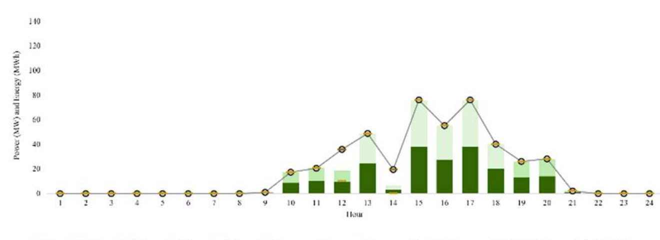

Figure 5. Energy traded by the photovoltaic power plant in the DA, ID, and tertiary markets and power traded in the secondary reserve market per hour

Performing the same type of analysis as for the price-maker scenario, from Fig. 4 and Fig. 5 it can be seen that:

- Hours where energy balance equals the total energy sold in the day-ahead and in the intraday markets:

  Between hours 9 to 11, 13, and 15 to 21, the generating company sells all produced energy exclusively in two markets. Specifically, during hours 9, 10, 11, 19, 20, and 21, energy is traded in intraday market 5, as it offers higher prices compared to other intraday sessions. In contrast, during hours 13, 15, 16, 17, and 18, intraday market 6 provides the most favorable prices. In these periods, selling energy in the other markets is not considered, as in some cases, for instance, the price of the upward tertiary reserve is lower than that of the downward reserve, effectively representing a cost. Additionally, the secondary reserve capacity market presents significantly lower prices than the intraday markets, further justifying this strategic choice.

- Hours where the energy balance is higher than the energy sold in the day-ahead and in the intraday markets

  During hours 12 and 14, the generating company adopts similar market strategies. In both cases, energy is sold in the day-ahead market as well as in the intraday session offering the most competitive price (intraday 5 at hour 12 and intraday 6 at hour 14). At hour 12, in addition to these markets, the generating company also sells upward energy in the tertiary reserve market, which presents the highest price, while avoiding selling downward energy. At hour 14, given the low prices in both the day-ahead and intraday markets, the generating company offsets this by increasing the selling of upward energy in the tertiary reserve market and also downward energy, despite the associated cost. In both hours, the generating company sells capacity in the secondary reserve market, which consistently offers the highest prices throughout daylight hours.

In this scenario, the generating company was modeled as a price-taker, implicitly assuming it has no influence on market prices. However, this assumption does not reflect the fact that by selling its production in the market, it does exert a downward influence on prices. To capture this effect, the market price was recalculated for each hour after the company's actions, considering the residual demand curve. These adjusted prices, reflecting the generating company's market impact, were then used to compute revenues in each market.

Fig. 6 illustrates the accumulated revenues per hour, including the cost of downward tertiary energy shown as a negative value. The effective total revenue for each hour is represented by the blue line.

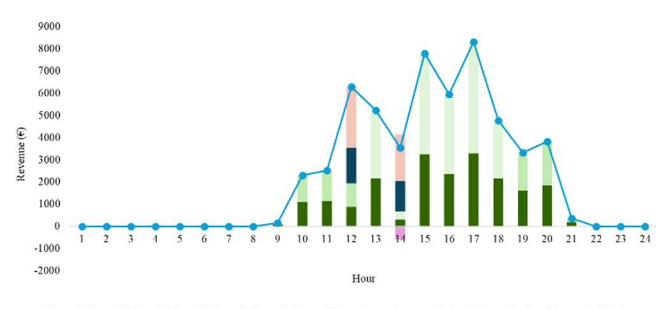

Figure 6. Revenue per market per hour

At hour 12, compared to other hours such as hours 13, 16, and 18, there is less PV production, but it shows a higher revenue. At this hour, although the generating company sells energy in the day-ahead and intraday markets, the tertiary reserve market represents the highest revenue because it has the highest price among all the markets. Selling downward energy in the tertiary reserve market is not considered, so there is no cost for this hour. Hour 14 shows a decrease in net revenue compared to the revenue from all markets because the generating company sells downward energy in the tertiary reserve market, which represents a cost. In this case, the generating company benefits from this cost as the increased revenue comes from the excess energy sold that cannot be put on the grid.

The total revenue for this day for the PV plant is 54361€.

IV. Conclusions

This study evaluates cross-market bidding strategies for a PV plant, including day-ahead, intraday, secondary reserve, and tertiary reserve markets, under price-maker and price-taker assumptions. This analysis contributes to the literature by providing a detailed view of the optimized management of energy assets across multiple markets.

The results show that in the price-maker scenario, plants achieve higher revenues. The price-maker scenario allows for better adaptation of offers to the market, thus increasing revenue.

The PV plant benefits more from intraday markets and has a limited impact on the price in a price-maker scenario. In a price-taker scenario, revenue decreases as the generating company optimizes the production across different markets, not assuming that its activity influences the final market price.

Just like the electrical system, electricity markets and production technologies are constantly evolving, allowing for continuous updates to this model. In future studies, it would be interesting to analyze other types of power plants, such as a PV power plant equipped with batteries. The inclusion of the continuous intraday market component or considering penalties for deviations in different markets could also be explored. Additionally, new programs and bidding rules, such as MARI, where offers are made in 15-minute intervals instead of one hour, shall be analyzed in future work.

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Appendix

A. Nomenclature

1) Indexes

c_m	Residual demand curve step index of the day-ahead and intraday markets for energy bought
i	Generating units' index
m	Day-ahead and intraday market index
t	Time period index
v_b	Residual demand curve step index of the secondary reserve capacity market
v_m	Residual demand curve step index of the day-ahead and intraday markets for energy sold

2) Sets

C_m	Set of residual demand curve step indexes of the day-ahead and intraday markets for energy bought
I	Set of generating units' indexes
M	Set of day-ahead and intraday market indexes
T	Set of time period indexes
V_b	Set of residual demand curve step index of the secondary reserve capacity market
V_m	Set of residual demand curve step index of the day-ahead and intraday markets for energy sold

3) Parameters

bc_{t,m,c_m}^{max}	Maximum energy of the residual demand curve step c_m, of market m, in period t, in MWh
bv_b_{t,v_b}^{max}	Maximum power of the residual demand curve step $v_$b, of the secondary reserve capacity market, in period t, in MW
bv_{t,m,v_m}^{max}	Maximum energy of the residual demand curve step v_m, of market m, in period t, in MWh

$c_{t,i}$ Operational costs of generating unit *i*, in period *t*, in €

GD_i Maximum ramp-down power of generating unit *i*, in MW

GS_i Maximum ramp-up power of generating unit *i*, in MW

$mFRR\_down_t$ Downward tertiary reserve energy required by the system operator, in period *t*, in MWh

$mFRR\_up_t$ Upward tertiary reserve energy required by the system operator, in period *t*, in MWh

$p_{t,i}^{max}$ Maximum output power of generating unit *i*, in period *t*, in MW

$p_{t,i}^{min}$ Minimum output power of generating unit *i*, in period *t*, in MW

$p_{t,PV}$ Output power of the PV power plant, in period *t*, in MW

p_{PV}^{max} Maximum output capacity of the PV power plant, in MW

$price\_down_t$ Price of the downward tertiary reserve energy, in period *t*, in €/MWh

$price\_up_t$ Price of the upward tertiary reserve energy, in period *t*, in €/MWh

$PVDATA_{t,PV}$ Unit power of the PV power plant, in period *t*, in MW

$qmin\_c_{t,m,c\_m}$ Sum of the energy of steps *1* to *c_m-1* of the demand curve of market *m*, in period *t*, in MWh

$qmin\_v\_b_{t,v\_b}$ Sum of the power of steps *1* to *v_b-1* of the demand curve of the secondary reserve capacity market, in period *t*, in MW

$qmin\_v_{t,m,v\_m}$ Sum of the energy of steps *1* to *v_m-1* of the demand curve of market *m*, in period *t*, in MWh

s_d Percentage of total energy traded in the day-ahead market relative to the total energy traded in the day-ahead and intraday markets.

sh_mFRR Maximum share of energy traded in the tertiary reserve market by the generating company

W^{max} Maximum storage capacity of the hydro unit, in MWh

W^{min} Minimum storage capacity of the hydro unit, in MWh

η_B Efficiency of the hydro unit while pumping

η_H Efficiency of the hydro unit while generating

$\lambda c_{t,\mathrm{m},c\_m}$ Price of the residual demand curve step *c_m*, of market *m*, in period *t*, in €/MWh

$\lambda v_{t,\mathrm{m},v\_m}$ Price of the residual demand curve step *v_m*, of market *m*, in period *t*, in €/MWh

$\lambda v\_b_{t,v\_b}$ Price of the residual demand curve step *v_b*, of the secondary reserve capacity market, in period *t*, in €/MW

4) Variables

bc_{t,m,c_m} Energy of the residual demand curve step *c_m*, of market *m*, in period *t*, in MWh

bv_b_{t,v_b} Power of the residual demand curve step *v_b*, of the secondary reserve capacity market, in period *t*, in MW

$bv_{t,\mathrm{m},v\_\mathrm{m}}$ Energy of the residual demand curve step *v_m*, of market *m*, in period *t*, in MWh

$f_{t,H}$ Binary variable that indicates if hydro unit is generating in period *t*.

$f_{t,B}$ Binary variable that indicates if hydro unit is pumping in period *t*.

$f_{t,i}$ Binary variable that indicates if generating unit *i* is operating in period *t*.

$p_{t,B}$ Consumption of the hydro unit while pumping, in period *t*, in MWh

$p_{t,H}$ Output power of the hydro unit while generating, in period *t*, in MWh

$p_{t,i}$ Total energy traded in the day-ahead, intradays and tertiary reserve market by the generating unit *i*, in period *t*, in MWh.

$p_{t,m,i}$ Total energy traded by generating unit *i*, in market *m*, in period *t*, in MWh

$p\_b_{t,i}$ Total power traded by the generating unit *i*, in the secondary reserve capacity market, in period *t*, in MW

${p_d}_{t,i}$ Energy traded in the day-ahead market by the generating unit *i*, in period *t*, in MWh

${p_{id1}}_{t,i}$ Energy traded in the first intraday market by generating unit *i*, in period *t*, in MWh

${p_{id2}}_{t,i}$ Energy traded in the second intraday market by generating unit *i*, in period *t*, in MWh

${p_{id3}}_{t,i}$ Energy traded in the third intraday market by generating unit *i*, in period *t*, in MWh

${p_{id4}}_{t,i}$ Energy traded in the fourth intraday market by generating unit *i*, in period *t*, in MWh

${p_{id5}}_{t,i}$ Energy traded in the fifth intraday market by generating unit *i*, in period *t*, in MWh

${p_{id6}}_{t,i}$ Energy traded in the sixth intraday market by generating unit *i*, in period *t*, in MWh

${p_{id7}}_{t,i}$ Energy traded in the seventh intraday market by generating unit *i*, in period *t*, in MWh

$p\_mFRR\_down_{t,i}$ Downward tertiary reserve energy provided by generating unit i, in period t, in MWh

$p\_mFRR\_up_{t,i}$ Upward tertiary reserve energy provided by generating unit i, in period t, in MWh

$qc_{t,m}$ Total energy bought by the generating company in market m, in period t, in MWh

qv_b_t Total power sold by the generating company in the secondary reserve capacity market, in period t, in MW

$qv_{t,m}$ Total energy sold by the generating company in market m, in period t, in MWh

$s_{t,i}$ Binary variable that indicates if generating unit i stopped in period t

uc_{t,m,c_m} Binary variable equal to 1 if step c_m is the marginal step of the residual demand curve of market m, in period t

uv_b_{t,v_b} Binary variable equal to 1 if step v_b is the marginal step of the residual demand curve of the secondary reserve capacity market, in period t

uv_{t,m,v_m} Binary variable equal to 1 if step v_m is the marginal step of the residual demand curve of market m, in period t

W_t Energy stored in the hydro unit in period t, in MWh

$y_{t,i}$ Binary variable that indicates if generating unit i started operating in period t.

π_{BS_t} Operational result of the secondary reserve capacity market, in period t, in €

π_{MD_t} Operational result of the day-ahead market, in period t, in €

π_{MIL_t} Operational result of the intraday market, in period t, in €

π_{RT_t} Operational result of the tertiary reserve market, in period t, in €

π_{Total} Operational profit of the generation company, in period t, in €

B. Mathematical formulation

The model used in this paper maximizes the operational profit over a period T, obtained by the difference between the total market revenues and the operational costs. The objective function is presented in (1).

$$max\,\pi_{Total} = \sum_{t=1}^{T}(\pi_{MD_t} + \pi_{MIL_t} + \pi_{BS_t} + \pi_{RT_t} - \sum_{i\in I} c_{t,i}) \qquad (1)$$

Where π_{MD_t}, π_{MIL_t}, π_{BS_t}, π_{RT_t} represent the operational result of the day-ahead, intraday, secondary reserve capacity, and tertiary reserve markets, respectively, in euros. The last term refers to the operational costs of each generation unit i, with $i \in \{H, B, T, PV\}$, where H corresponds to a hydroelectric power plant when generating, B to a hydroelectric power plant when pumping, T to a thermal power plant, and PV to a photovoltaic plant.

For the price-maker scenario, the model draws upon the residual demand curve that the generating company faces [7]. Fig. 7 represents the residual demand curve with the notation of the model, for the case where the generating company sells energy in the market, while Fig. 8 represents the same residual demand curve when the generating company purchases energy in the market.

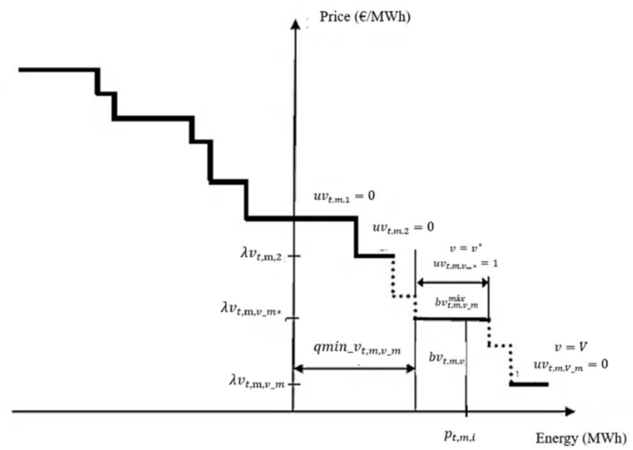

Figure 7. Residual demand curve when the generating company sells energy in the market

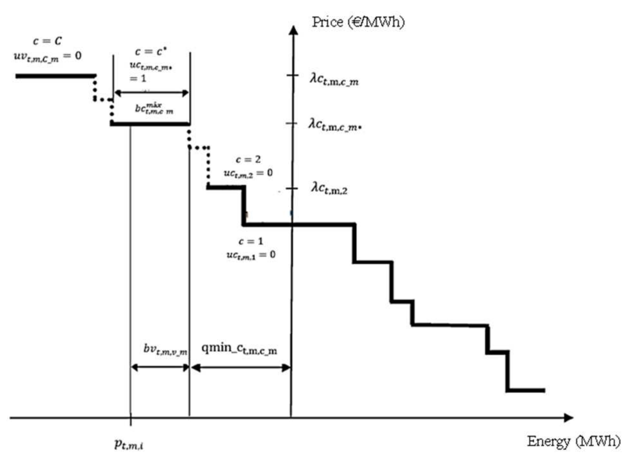

Figure 8. Residual demand curve when the generating company purchases energy in the market

The residual demand curves depicted in Fig. 7 and Fig. 8 model the generating company's behavior in the day-ahead and in the various sessions of the intraday market. For the secondary reserve capacity market, since the capacity offered is always a positive value, the residual demand curve of the generating company in that market is analogous to the one depicted in Fig. 7.

For better understanding of (1), each term was subdivided by market. The operational result of the day-ahead market is presented in (2):

$$\pi_{MD_t} = \sum_{v\_m=1}^{V\_m} \lambda v_{t,m,v\_m}\big(bv_{t,m,v\_m} + uv_{t,m,v\_m}\, qmin\_v_{t,m,v\_m}\big) + \sum_{c\_m=1}^{C\_m} \lambda c_{t,m,c\_m}\big(bc_{t,m,c\_m} + uc_{t,m,c\_m} qmin\_c_{t,m,c\_m}\big) \quad (2)$$

The first term of (2) calculates the operational result of sales, while the second refers to the operational result of purchases.

The operational result of the intraday market is given by (3):

$$\pi_{MIL_t} = \sum_{\substack{m \\ m \neq d}}^{M} \left(\sum_{\substack{v\_m \\ v\_m \neq v\_d}}^{V\_m} \lambda v_{t,m,v\_m}\big(bv_{t,m,v\_m} + uv_{tm,,v\_m}\, qmin\_v_{t,m,v\_m}\big) + \sum_{\substack{c\_m \\ c\_m \neq c\_d}}^{C\_m} \lambda c_{t,m,c\_m}\big(bc_{t,m,c\_m} + uc_{t,m,c\_m}\, qmin\_c_{t,m,c\_m}\big) \right) \quad (3)$$

The operational result of the secondary reserve market is given by (4):

$$\pi_{BS_t} = \sum_{v\_b=1}^{V\_b} \lambda v\_b_{t,v\_b}\big(bv\_b_{t,v\_b} + uv\_b_{t,v\_b}\, \mathrm{qmin\_v\_b}_{t,v\_\mathrm{b}}\big) \quad (4)$$

Finally, the operational result of the tertiary reserve market:

$$\pi_{RT_t} = \sum_{i \in I} (p\_mFRR\_up_{t,i} * price\_up_t - p\_mFRR\_down_{t,i} * price\_down_t) \quad (5)$$

Equation (5) expresses the operational result of the tertiary reserve, where the first term represents the revenue from the energy provided for upward regulation, while the second term corresponds to the energy for downward regulation, which represents a cost for the generating company.

It should be noted that the generating company's behavior is always modeled as a price-taker in the tertiary reserve market, this is the residual demand curves shown in Fig. 7 and Fig. 8 do not apply to this market.

1) Constraints Based on the Residual Demand Curve

Based on the residual demand curves and the characteristics of each market, constraints were imposed on (1) to (5) ensuring that the results are as realistic as possible.

$$qv_{t,m} = \sum_{v\_m=1}^{V\_m} \big(bv_{t,m,v\_m} + uv_{t,m,v\_m}\, qmin\_v_{t,m,v\_m}\big) \quad (6)$$

$$qv\_b_t = \sum_{v\_b=1}^{V\_b} \big(bv\_b_{t,v\_b} + uv\_b_{t,v\_b}\, qmin\_v\_b_{t,v\_b}\big) \quad (7)$$

Equation (6) calculates the total energy sold by the generating company in the day-ahead market and in each session of the intraday market. Equation (7) determines the capacity sold in the secondary reserve market.

$$qc_{t,m} = \sum_{c\_m=1}^{C\_m} \big(bc_{t,m,c\_m} + uc_{t,m,c\_m}\, qmin\_c_{t,m,c\_m}\big) \quad (8)$$

Equation (8) calculates the total amount of energy purchased by the generating company in the day-ahead and intraday markets.

$$\sum_{i \in I} p_{t,m,i} = qv_{t,m} + qc_{t,m} \quad (9)$$

$$\sum_{i \in I} p\_b_{t,i} = qv\_b_t \quad (10)$$

Equation (9) ensures that, at each instant t, the total energy sold and purchased in the day-ahead market or in each intraday session is equal to the sum of the energy traded by each unit i of the generating company. Equation (10) applies the same principle to the secondary reserve market.

$$\sum_{i \in I} p\_b_{t,i} \leq \sum_{v\_b=1}^{V\_b} bv\_b_{t,v\_b}^{max} \quad (11)$$

Equation (11) indicates that the sum of the capacity sold by all units i of the generation company in the secondary reserve market must not exceed the total capacity required by the system operator.

$$\sum_{v\_m=1}^{V\_m} uv_{t,m,v\_m} + \sum_{c\_m=1}^{C\_m} uc_{t,m,c\_d} = 1 \quad (12)$$

$$\sum_{v\_b=1}^{V\_b} uv\_b_{t,v\_b} = 1 \quad (13)$$

Equation (12) ensures that only one binary variable is different from zero, this is, that a single step of the residual demand curve in the day-ahead market and in each intraday market session is active, whether for buying or selling. Equation (13) ensures the same for the secondary reserve capacity market.

$$0 \leq bv_{t,m,v\_m} \leq uv_{t,m,v\_m} bv_{t,m,v\_m}^{max} \quad (14)$$

$$0 \leq bv\_b_{t,v\_b} \leq uv\_b_{t,v\_b} bv\_b_{t,v\_b}^{max} \quad (15)$$

Equation (14) requires that the quantity sold of the active step of the residual demand curve does not exceed the total quantity of that step. Equation (15) applies the same reasoning to the secondary reserve capacity market.

$$-uc_{t,m,c\_m} bc_{t,m,c\_m}^{max} \leq bc_{t,m,c\_m} \leq 0 \quad (16)$$

In (16), the quantity bought of the active step of the residual demand curve does not exceed the total quantity of the step. Negative values represent the fact that when energy is bought it is considered a negative value.

2) Additional Equations

$$p_{t,PV} = PVDATA_{t,PV} \times p_{PV}^{max} \quad (17)$$

Equation (17) calculates the power produced in each period t by a PV power plant, where the first term refers to the production of a 1 MW plant in period t. The last term refers to the power rating of the power plant under analysis.

To increase the realism of the model, it was necessary to impose a minimum quota in the day-ahead market, as demonstrated in condition (18).

$$p_{d_{t,i}} \geq \frac{s\_d}{1 - s\_d} (p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} + p_{id4_{t,i}} + p_{id5_{t,i}} + p_{id6_{t,i}} + p_{id7_{t,i}}) \quad (18)$$

The parameter s_d represents the minimum percentage allocated to the day-ahead market relative to the total energy traded in the day-ahead and intraday markets. The inclusion of this equation, forces that a minimum quantity of energy must be sold in the day-ahead market, thus preventing, for instance, that the energy produced by a generating unit i is only sold in the intraday markets.

$$p_{t,i} = p\_d_{t,i} + p\_id1_{t,i} + p\_id2_{t,i} + p\_id3_{t,i} + p\_id4_{t,i} + p\_id5_{t,i} + p\_id6_{t,i} + p\_id7_{t,i} + p\_mFRR\_sub_{t,i} - p\_mFRR\_desc_{t,i} \quad (19)$$

Equation (19) calculates the total energy sold by a generating unit i in the day-ahead, intraday, and tertiary reserve markets.

$$p_{t,i} - p\_mFRR\_up_{t,i} + p\_mFRR\_down_{t,i} + \frac{2}{3} p\_b_{t,i} \leq p_{t,i}^{max} \quad (20)$$

$$p_{t,i} - p\_mFRR\_up_{t,i} + p\_mFRR\_down_{t,i} - \frac{1}{3} p\_b_{t,i} \geq p_{t,i}^{min} \quad (21)$$

Equations (20) and (21) reflect the physical limitations of the power plants to ensure their correct operation. Restrictions are imposed on the maximum power $p_{t,i}^{máx}$ and minimum power $p_{t,i}^{min}$. For the generating units participating in the secondary reserve capacity market, one-third of the sold secondary reserve capacity must be available for downward regulation and two-thirds for upward regulation. The one-third and the two-third shares stem from the Portuguese rules for the secondary reserve market by the time of this study.

$$0 \leq \sum_{i \in I} p\_mFRR\_up_{t,i} * sh\_mFRR \leq mFRR\_up_t \quad (22)$$

$$0 \leq \sum_{i \in I} p\_mFRR\_down_{t,i} * sh\_mFRR \leq mFRR\_down_t \quad (23)$$

In the tertiary reserve, energy and price data were used to define parameters such as the maximum upward and downward energy required by the system operator. The variables $p\_mFRR\_\mathrm{up}_{t,i}$ and $p\_mFRR\_down_{t,i}$ represent the energy sold by the generating company in each period t and generating unit i which maximize operational profit. Equations (22) and (23) specify that these variables must be positive and not exceed the maximum amount of tertiary reserve required by the operator. A scalar, sh_mFRR , was added to limit the share of upward and downward energy sales in the tertiary reserve market by the generating company.

$$p_{d_{t,i}} \leq p_{t,i}^{max} \quad (24)$$

$$p_{d_{t,i}} \geq p_{t,i}^{min} \quad (25)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} \leq p_{t,i}^{max} \quad (26)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} \geq p_{t,i}^{min} \quad (27)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} \leq p_{t,i}^{max} \quad (28)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} \geq p_{t,i}^{min} \quad (29)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} \leq p_{t,i}^{max} \quad (30)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} \geq p_{t,i}^{min} \quad (31)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} + p_{id4_{t,i}} \leq p_{t,i}^{max} \quad (32)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} + p_{id4_{t,i}} \geq p_{t,i}^{min} \quad (33)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} + p_{id4_{t,i}} + p_{id5_{t,i}} \leq p_{t,i}^{max} \quad (34)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} + p_{id4_{t,i}} + p_{id5_{t,i}} \geq p_{t,i}^{min} \quad (35)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} + p_{id4_{t,i}} + p_{id5_{t,i}} + p_{id6_{t,i}} \leq p_{t,i}^{max} \quad (36)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} + p_{id4_{t,i}} + p_{id5_{t,i}} + p_{id6_{t,i}} \geq p_{t,i}^{min} \quad (37)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} + p_{id4_{t,i}} + p_{id5_{t,i}} + p_{id6_{t,i}} + p_{id7_{t,i}} \leq p_{t,i}^{max} \quad (38)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} + p_{id4_{t,i}} + p_{id5_{t,i}} + p_{id6_{t,i}} + p_{id7_{t,i}} \geq p_{t,i}^{min} \quad (39)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} + p_{id4_{t,i}} + p_{id5_{t,i}} + p_{id6_{t,i}} + p_{id7_{t,i}} + p_{mFRRsub_{t,i}} \leq p_{t,i}^{max} \quad (40)$$

$$p_{d_{t,i}} + p_{id1_{t,i}} + p_{id2_{t,i}} + p_{id3_{t,i}} + p_{id4_{t,i}} + p_{id5_{t,i}} + p_{id6_{t,i}} + p_{id7_{t,i}} - p_{mFRRdesc_{t,i}} \geq p_{t,i}^{min} \quad (41)$$

To optimize each market without exceeding the maximum energy, conditions (24) to (41) were used. These constraints allow for the analysis and optimization of each period successively. In equations (40) and (41), in addition to the day-ahead and intraday markets, the tertiary reserve market was also considered. Specifically, condition (40) included the tertiary reserve energy for upward regulation, while equation (41) subtracted the tertiary reserve energy for downward regulation from the day-ahead and intraday markets.

Given that some intraday markets have distinct starting times than the first hour, (42) to (45) were applied to ensure that for time periods prior to the starting hour of these markets, the energy traded is zero.

$$p_{id4_{t<4,i}} = 0 \quad (42)$$

$$p_{id5_{t<7,i}} = 0 \quad (43)$$

$$p_{id6_{t<12,i}} = 0 \quad (44)$$

$$p_{id7_{t<20,i}} = 0 \quad (45)$$

3) Physical Unit Restrictions

$$p_{t,i} - p_{t-1,i} \leq GS_i \quad (46)$$

$$p_{t-1,i} - p_{t,i} \leq GD_i \quad (47)$$

In constraints (46) and (47), maximum ramp-up power (GS_i) and maximum ramp-down power (GD_i) are considered. Condition (46) stipulates that the change in power of a generation unit between two periods should not exceed the ramp-up power (GS_i), while condition (47) limits the downward change to the ramp-down power (GD_i).

$$y_{t,i} - s_{t,i} = f_{t,i} - f_{t-1,i} \quad (48)$$

Equation (48) describes the operation, start, and stop of each generation unit i in each period t. The binary variable $f_{t,i}$ indicates whether the unit is operating, $y_{t,i}$ indicates whether it has started production, and $s_{t,i}$ indicates whether it has stopped production.

$$f_{t,H} + f_{t,B} \leq 1 \quad (49)$$

Equation (49) applies to a pumped storage hydroelectric plant to ensure that it does not operate both the turbine and the pump simultaneously.

$$W_t = \begin{cases} W_{t-1} - \eta_B p_{t,B} \;\; if \; p_{t,B} < 0 \\ W_{t-1} - \dfrac{p_{t,H}}{\eta_H} \;\;\; if \; p_{t,H} \geq 0 \end{cases} \quad (50)$$

Equation (50) calculates the stored energy W_t in the reservoir of the pumped storage hydroelectric plant, depending on the plant's operational mode.

$$W^{min} \leq W_t \leq W^{max} \quad (51)$$

Equation (51) ensures that the energy W_t stored in the reservoir of the pumped storage hydroelectric plant must be between the minimum energy W^{min} and the maximum energy W^{max} defined for the reservoir.