Structural Program of Electricity Demand Response in Brazil: Bidding Strategy and Market Equilibria

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***Abstract*—In this paper, we evaluate the Brazilian Demand Response (DR) initiative called Structural Program (SP) from the perspective of both consumers and market-design agents. We begin by characterizing the operational and financial dynamics of the SP, proposing a methodology for devising bidding strategies for consumers in the program as a two-stage problem modeled in a bilevel optimization structure. More specifically, the first stage aims to maximize the consumer benefits by optimizing bid parameters and the second stage evaluates market and system dynamics to assess the bids clearance in the day-ahead dispatch program. Then, a procedure to identify Nash Equilibrium points is described based on fixed-point techniques. A numerical experiment using real data from the Brazilian power system is presented to discuss insights of the Structural Program under a Nash Equilibrium point. We illustrate that this DR initiative can enhance participant outcomes, improve market efficiency, and strengthen system flexibility.**

***Index Terms*—Demand Response, Energy Economics, Multi-Level Optimization, Optimal Bidding, Nash Equilibrium**

I. Introduction

OVER the past decades, power systems worldwide have experienced a significant progression in operational uncertainty driven by several factors, including, on the one hand, the widespread integration of centralized and distributed Renewable Energy Sources (RESs), and a substantial increase in the connection of electric vehicles and data centers to the grid, the latter fueled by advancements in artificial intelligence and the cryptocurrency sector, adding a new layer of unpredictability to electricity consumption patterns, on the other hand [1]. In this context, novel investments in grid infrastructure and generation capacity are frequently deemed mandatory to enhance system flexibility and enable operators to maintain supply-demand balance, especially during peak-load periods [2], [3]. When devising options to introduce new operational flexibility for system operators, however, the energy consumption is typically regarded as unresponsive (inelastic) to the system conditions [4]–[6]. Nevertheless, Demand-Side Management (DSM) initiatives, particularly Demand Response (DR) programs, are consensual among policymakers and market participants as potential solutions for providing operators with high system flexibility (by shifting or shedding electricity consumption) akin to physical energy storage. In fact, they are often recognized as the most cost-effective alternative to grid and capacity expansion, offering significant potential to enhance market efficiency, particularly within the ongoing clean energy transition [7]–[9]. Across the globe, numerous DR programs have been discussed and implemented over the past decade [10]. Overall, the programs vary in design, operations, and payment methods, ranging from offering reimbursements to commercial and residential consumers in exchange for direct control of their appliances (e.g., air conditioning) to different pricing and capacity payment mechanisms [11]. In general, DR programs are classified into two main categories: Price-Based Programs (PBP) and Incentive-Based Programs (IBP). In PBPs, consumers voluntarily reduce their load in response to economic signals, such as dynamic pricing or time-of-use tariffs. Conversely, IBPs offer customers direct financial incentives to achieve specific load alterations within a designated time frame or trigger [12].

Particularly across Europe, following the European Commission's Clean Energy Package launched in 2016, Member States are progressing in various directions, such as defining the role of aggregators and allowing their participation in DR programs, revamping market structures and payment mechanisms, and creating pathways for new products to become accessible to a broader range of consumers [13]. In fact, in countries where DR initiatives have historically been nearly absent, such as Estonia, Spain, and Italy, there is now growing interest in exploring their potential. Conversely, France and Great Britain are recognized for having the most advanced DR frameworks [14]. More specifically, since 2003, consumers in France have actively participated in electricity markets, with large industrial customers initially pioneering participation through the balancing mechanisms, followed by residential aggregators in 2007. Nowadays, demand-side participation in France spans various markets, including Day-Ahead, Intraday, FCR (Frequency Containment Reserve), aFRR (Automatic Frequency Restoration Reserve), and others. Notably, we highlight the *Notification d'Échanges de Blocs d'Effacement* (NEBEF) mechanism, which allows all consumption sites in mainland France, as self-represented entities or through third-party aggregators, to participate by providing demand response in exchange for remuneration in energy markets, either through over-the-counter agreements or via day-ahead and intraday power exchanges. This is achieved on fair and equal terms with generation companies without requiring supplier consent [15]. Great Britain was the first country in Europe to open several electricity markets to consumer participation. Today, nearly all balancing services are accessible to demand response, and the capacity mechanism also allows consumer participation, although not on terms equivalent to those for generation companies. In contrast, both the balancing mechanism and wholesale markets are closed to aggregators [16]. In recent years, several design challenges in Great Britain's demand response framework have been widely debated. Key issues include disputes between aggregators and Balancing Responsible Parties (BRPs) within the balancing mechanism and wholesale markets, inconsistencies in measurement and baseline methodologies, and complexities in bidding processes. These obstacles have been pointed out as key factors for a decline in demand-side participation, limiting its potential for flexibility and efficiency [17].

In the particular context of the Brazilian power system and

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market environments, initiatives to promote DSM and DR are still in their early stages [18], [19]. Historically, consumers did not play an active role in the operation of the Brazilian power system until 2018, when the so-called *Pilot DR Program* was launched [20]. This program allowed the system operator to use load reduction bids from consumers as an alternative to dispatching conventional thermoelectric generators. However, the Pilot DR Program officially ended in mid-2022 due to several design issues and adverse market conditions that resulted in low participation rates. Nevertheless, in 2020–2021, Brazil faced severe hydrological conditions, motivating the creation of an emergency DR program called *Voluntary Load Reduction* (VLR) [20]. Built upon the previous Pilot DR Program, this initiative incorporated several key improvements, including enabling aggregators to participate in the program, eliminating the need for signing a contract to provide such ancillary services, and removing the penalty for exceeding network usage during compensation, resulting in more effective consumer participation. The success of this initiative led to the formalization of this emergency program as a *Structural Program* (SP) in 2022, which operates through a bid-based mechanism and is currently active in Brazil [21].

In this work, we evaluate the SP initiative for demand response from the perspective of a consumer and a market-design agent. For this purpose, we first model the operational dynamics of the SP and propose a methodology to devise bidding strategies for consumers. The methodology frames the decision-making process as a two-stage problem modeled in a bilevel optimization structure [22], [23]. More specifically, in the first stage, we focus on maximizing the consumer's benefits by determining optimal bid parameters accounting for the opportunity cost of reducing its energy consumption. The second stage examines market and system dynamics, particularly the interactions with the Brazilian day-ahead dispatch scheduling program, to evaluate if the system operator should clear the bid. Then, by leveraging the concept of Nash Equilibrium [24], [25], we estimate the behavior of consumers with distinct characteristics — emulating various sectors of the economy — participating in the SP and analyze market equilibrium states. Through these advanced modeling techniques, on the one hand, market participants can devise optimal bidding strategies in this program, and, on the other hand, regulatory and market-design agents are able to assess the value and the characteristics of the program in place, potentially adjusting some of its design aspects. To illustrate the applicability of these bidding and equilibrium methodologies, we present a numerical experiment based on the Brazilian power system and market environment. The case study highlights how optimized bidding strategies can enhance participant outcomes, improve market efficiency, and increase system flexibility. Our findings suggest that a more structured bidding approach could significantly enhance the effectiveness of DR programs in Brazil, providing substantial benefits to both participants and the energy system as a whole.

II. Demand Response: The Structural Program (SP)

Since 2022, Brazil has implemented a demand response initiative named *Structural Program* [21]. It is destined for a particular class of consumers with high load levels, referred to as *self-represented*, or for aggregators, a third party responsible for acting in this initiative on behalf of consumers. Although the Brazilian power system operates in a cost-based arrangement [19], the SP is implemented through a bid-based mechanism in which the allowed participants (large consumers or aggregators) bid a maximum volume reduction at a given market zone along with a minimum compensation price. The auctions take place on a weekly basis, whose products are specified by the Brazilian system operator based on the expected need for next week's operations. Bid submission closes on Thursdays at noon. Consumers who have submitted bids in the weekly auction have the option to indicate to the system operator their availability for load modulation in the next day's operation, one day in advance. The typical setting is to auction products with a block duration of 4 to 17 hours for each weekday and Saturday. Therefore, there are products for each day of the following operative week (Saturday to Friday, excluding Sunday) and system areas, with a duration of 4 to 17 hours.

Formally, let $\mathcal{J}$ be the set of days within the week with a product to be auctioned (e.g., weekdays and Saturday) with $\mathcal{H}(j)$ the respective index set of hours and $\overline{\mathcal{H}}(j) \subset \mathcal{H}(j)$ the block duration (subset of hours) of the product to be auctioned for the day $j \in \mathcal{J}$. Furthermore, we assume a collection $\mathcal{I} = \{1, \ldots, n\}$ of n consumers willing to compete in the SP auction for the next week, with each consumer $i \in \mathcal{I}$ bidding a minimum compensation price $\lambda_{i,j}$ and a maximum volume reduction $V_{i,j}$ for each day $j \in \mathcal{J}$. Each consumer $i \in \mathcal{I}$ is assumed to have a total consumption given by $d_{i,h} = d^{(F)}_{i,h} + d^{(V)}_{i,h}$, decomposed into a fixed $d^{(F)}_{i,h}$ and a controllable $d^{(V)}_{i,h}$ load for each hour $h \in \mathcal{H}(j)$ of the day $j \in \mathcal{J}$, the latter used to modulate the consumer's total demand to balance its primary operations and the demand response requirements. In the SP, the reduction countability for the consumer $i \in \mathcal{I}$ is determined using a baseline load $L_{i,h}$, *a priori* specified for each hour $h \in \mathcal{H}(j)$ of $j \in \mathcal{J}$, calculated using historical data [26]. Then, for a given consumer $i \in \mathcal{I}$, the amount of measured reduction in consumption in the block $h \in \overline{\mathcal{H}}(j)$ of $j \in \mathcal{J}$ is given by

$$B\big(d_{i,h}\big) = \max\Big\{L_{i,h} - d_{i,h}, 0\Big\}. \tag{1}$$

On the other hand, the amount actually cleared ($q_{i,j}$) by the consumer $i \in \mathcal{I}$ in the SP auction for the day $j \in \mathcal{J}$ follows the solution of a least-cost day-ahead operation [27], [28]. Therefore, the amount to be compensated by consumer $i \in \mathcal{I}$ in the block $h \in \overline{\mathcal{H}}(j)$ of $j \in \mathcal{J}$ is denoted by

$$Q\big(d_{i,h}, q_{i,j}\big) = \min\Big\{B\big(d_{i,h}\big), q_{i,j}\Big\}. \tag{2}$$

Finally, SP auctions resemble a pay-as-bid mechanism, with the clearing price for the consumer $i \in \mathcal{I}$ in each $h \in \overline{\mathcal{H}}(j)$ of $j \in \mathcal{J}$ given by the largest value among the price bid ($\lambda_{i,j}$) and the energy spot price (π_h). Therefore, for each $h \in \overline{\mathcal{H}}(j)$ of $j \in \mathcal{J}$, the revenue of the consumer $i \in \mathcal{I}$ in the SP is given by

$$R\Big(d_{i,h}, q_{i,j}, \lambda_{i,j}, \pi_h\Big) = Q\big(d_{i,h}, q_{i,j}\big) \max\Big\{\lambda_{i,j}, \pi_h\Big\}. \tag{3}$$

Remark 1. *It should be noted that a penalty is applied to a consumer $i \in \mathcal{I}$ whenever its total consumption, $d_{i,h}$, exceeds, in a given hour within the block duration $h \in \overline{\mathcal{H}}(j)$, an upward tolerance margin over the baseline consumption (e.g., demand higher than 110% of the baseline consumption of hour h). This penalty is essentially a reduction in the amount of energy $Q(\cdot,\cdot)$ that can be compensated for in the SP and is proportional to the volume that exceeds this threshold. In this work, we do not explicitly include this penalty in* (3)*. Instead, we impose a hard constraint that requires that the total consumption during $\overline{\mathcal{H}}(j)$ be less than the penalty threshold.*

Next, we formally outline the consumer's optimal behavior in the Structural Program as a bi-level optimization model. We focus on self-represented consumers but highlight that extending the methodology to aggregators is relatively straightforward since most of the principles of the SP are also equivalent to aggregators.

III. Optimal Bidding Problem in the SP

In this section, we characterize the bidding model of a self-represented consumer in the Brazilian SP as a bi-level optimization problem. For expository purposes, we assume a single-node network; consequently, the bids are for the same market zone. Furthermore, we also consider that each consumer $i \in \mathcal{I}$ has an opportunity cost functional $\Theta(d_{i,h})$ that recovers its cost of load consumption, thus valuing the load reduction. The proposed bidding model for a given self-represented consumer $i \in \mathcal{I}$ is presented next.

$$\max_{\substack{\boldsymbol{\lambda}_i, \boldsymbol{V}_i, \boldsymbol{q}_i, \\ \boldsymbol{d}_i, \boldsymbol{d}_i^{(V)}}} \sum_{j \in \mathcal{J}} \left(\sum_{h \in \overline{\mathcal{H}}(j)} R\Big(d_{i,h}, q_{i,j}, \lambda_{i,j}, \pi_h\Big) - \sum_{h \in \mathcal{H}(j)} \Theta\big(d_{i,h}\big) \right) \quad (4)$$

subject to:

$$0 \leq d_{i,h} \leq \big(1 + \gamma\big) L_{i,h}, \qquad \forall\, h \in \overline{\mathcal{H}}(j),\ j \in \mathcal{J}; \quad (5)$$

$$d_{i,h} = d_{i,h}^{(F)} + d_{i,h}^{(V)}, \qquad \forall\, h \in \mathcal{H}(j),\ j \in \mathcal{J}; \quad (6)$$

$$\big(\lambda_{i,j} \times V_{i,j}\big) \in \mathbb{R}_+ \times \big[\underline{V}_i, \overline{V}_i\big], \qquad \forall\, j \in \mathcal{J}; \quad (7)$$

$$q_{i,j} \in \arg\min_{\boldsymbol{g}, \boldsymbol{q}} \left\{ \sum_{h \in \mathcal{H}(j)} \sum_{k \in \mathcal{K}} c_{k,h} g_{k,h} + \sum_{h \in \overline{\mathcal{H}}(j)} \sum_{i' \in \mathcal{I}} \lambda_{i',j} q_{i',j} \right. \quad (8)$$

subject to:

$$\sum_{k \in \mathcal{K}} g_{k,h} + \sum_{i' \in \mathcal{I}} q_{i',j} = \hat{D}_h, \qquad \forall\, h \in \overline{\mathcal{H}}(j); \quad (9)$$

$$\sum_{k \in \mathcal{K}} g_{k,h} = \hat{D}_h, \qquad \forall\, h \in \mathcal{H}(j) \setminus \overline{\mathcal{H}}(j); \quad (10)$$

$$0 \leq g_{k,h} \leq G_{k,j}, \qquad \forall\, k \in \mathcal{K},\ h \in \mathcal{H}(j); \quad (11)$$

$$\left. 0 \leq q_{i',j} \leq V_{i',j}, \qquad \forall\, i' \in \mathcal{I}; \right\}, \ \forall\, j \in \mathcal{J}. \quad (12)$$

Problem (4)–(12) falls into the class of a bi-level optimization model [22]. The first level aims to co-optimize the price and volume bidding into the SP with the consumer's $i \in \mathcal{I}$ demand level in each hour of the following week such that the revenue $R(\cdot)$ compensated by the opportunity cost $\Theta(\cdot)$ is maximized. Constraint (5) limits the total consumption along the block duration $\overline{\mathcal{H}}(j)$ in each day $j \in \mathcal{J}$ to the penalty threshold $(1+\gamma)L_{i,h}$, with $\gamma > 0$, following the discussion in **Remark 1**. Furthermore, the block of constraints (7) specifies the support of the price and volume bidding variables, with the volume bid assumed to be bounded and $0 \leq \underline{V}_i \leq \overline{V}_i$ as lower- and upper-bounds[1]. The lower-level problem (8)–(12) emulates the clearing process. Structurally, it is an economic dispatch problem [27], [28] that aims to minimize the operational cost for meeting demand in every hour of the day $j \in \mathcal{J}$ accounting for the price bid $\lambda_{i,j}$ from all consumers $i \in \mathcal{I}$ for the day $j \in \mathcal{J}$. In (8), $c_{k,h}$ stands for the hourly marginal production cost of the energy supplier $k \in \mathcal{K}$ and $g_{k,h}$ is the respective generation. Constraints (9) and (10) represents the energy balance constraint with $\hat{D}_h$ the expected overall system load in hour $h \in \mathcal{H}(j)$ of day $j \in \mathcal{J}$. Furthermore, constraint (11) specifies the support of the generation variable $g_{k,h}$ with $G_{k,j}$ the production capacity of the energy supplier $k \in \mathcal{K}$ in day $j \in \mathcal{J}$ and constraint (12) bounds the quantity[2] $q_{i,j}$ cleared by the consumers $i \in \mathcal{I}$ in day $j \in \mathcal{J}$ to the volume bid $V_{i,j}$. In Brazil, the power system operates under a cost-based structure, where day-ahead scheduling is determined by a complex security-constrained unit commitment problem [29], [30]. The resulting dispatch is subsequently adjusted to account for operational aspects not explicitly represented in the model. In real-time operation, load variations are primarily addressed through the re-dispatch of hydroelectric generators by the system operator, as there is no official balancing market in place.

Two points worth highlighting. Firstly, unlike several Demand Response initiatives around the globe, the SP auction and the respective clearing amounts do not impact the energy spot pricing in Brazil. Structurally, the energy spot price in Brazil is calculated by the market operator as the dual variable of the energy balance constraint, similar to most energy markets worldwide [27], [31]. Therefore, the connection between the energy spot pricing pipeline and the SP auction clearing process is that they consider the same system conditions, but the official production cost model for spot pricing does not account for the demand response flexibility provided by the SP. In the context of this work, for the purpose of defining the optimal bidding for consumers, the energy spot price that impacts the revenue (3) is an exogenous variable. Secondly, problem (4)–(12) is a bi-level model with a mixed-integer non-linear programming problem as the upper level and a linear and continuous programming problem as the lower level. In the past decades, several solution methods have been studied in the technical literature to handle non-convex optimization problems and specific classes of bi-level programs [32], [33]. In the following subsection, we leverage these techniques to present a procedure to reformulate the proposed bidding model (4)–(12) as a tractable single-level optimization problem.

A. Single-Level Equivalent Formulation

The bidding problem (4)–(12) has two sources of non-tractability. The first one is its bi-level structure composed of a two-level system of hierarchical mathematical programming problems [22]. In the past decades, several techniques to handle this class of problems have been discussed in the technical literature, particularly the ones with a linear and continuous lower-level model. The most common approach is to characterize the optimality set of the lower-level problem through its Karush-Kuhn-Tucker (KKT) necessary conditions [34], [35] and replace the entire lower-level problem with them. Then, the complementarity constraints are handled using the so-called Fortuny-Amat technique [36], which re-writes the bilinear constraints as a mixed-integer linear set of restrictions. As a result, this reformulation procedure essentially replaces the lower-level problem (8)–(12) by a collection of mixed-integer linear constraints[3].

The second source of non-tractability stems from the definition of the revenue function (3). Firstly, for a given consumer $i \in \mathcal{I}$ and block duration $h \in \overline{\mathcal{H}}(j)$ of $j \in \mathcal{J}$, an auxiliary variable $\alpha_{i,h}$ recovers the amount of measured reduction in consumption, $B\big(d_{i,h}\big)$, i.e., $\alpha_{i,h} = B\big(d_{i,h}\big) = \max\big\{L_{i,h} - d_{i,h}, 0\big\}$, if $\exists\, \nu_{i,h} \in \{0,1\}$ such that

$$\begin{cases} L_{i,h} - d_{i,h} \leq \alpha_{i,h} \leq L_{i,h} - d_{i,h} + M^{(1)}\big(1 - \nu_{i,h}\big) \\ 0 \leq \alpha_{i,h} \leq M^{(1)} \nu_{i,h} \end{cases},$$

with $M^{(1)}$ a sufficiently large number to relax the constraints. Then, for a given consumer $i \in \mathcal{I}$ and block duration $h \in \overline{\mathcal{H}}(j)$

[1]Some SP auctions enforce a regulatory minimum volume bid (e.g., $\underline{V} = 5$ MW) for all participating consumers and require that volume bids be submitted as integer values. In this work, we relax the integrality constraint.

[2]We note that, while not a formal rule, the Brazilian system operator typically enforces an *all-or-nothing* clearing practice, i.e., a consumer's bid is either fully accepted or entirely rejected. In this work, we soft this practice by allowing bids to be partially cleared.

[3]We refer to [23], [25], [37], [38] and the references therein for further details on this reformulation with applications in strategic bidding/offering in electricity markets.

of $j \in \mathcal{J}$, the optimal value of the following bilinear problem recovers the revenue function (3).

$$R\Big(d_{i,h}, q_{i,j}, \lambda_{i,j}, \pi_h\Big) = \min\Big\{\alpha_{i,h}, q_{i,j}\Big\} \max\Big\{\lambda_{i,j}, \pi_h\Big\}$$
$$= \max_{r_{i,h}, \gamma_{i,h}} r_{i,h}$$
subject to:
$$r_{i,h} \leq q_{i,j}\pi_h + M^{(2)}\big(1-\gamma_{i,h}\big);$$
$$r_{i,h} \leq \alpha_{i,h}\pi_h + M^{(2)}\big(1-\gamma_{i,h}\big);$$
$$r_{i,h} \leq q_{i,j}\lambda_{i,j} + M^{(2)}\gamma_{i,h};$$
$$r_{i,h} \leq \alpha_{i,h}\lambda_{i,j} + M^{(2)}\gamma_{i,h};$$
$$\gamma_{i,h} \in \{0,1\}.$$

with $M^{(2)}$ a sufficiently large number to relax the constraints. Therefore, the bidding model (4)–(12) can be equivalently written as the single-level optimization problem in (13)–(23).

$$\max_{\substack{\boldsymbol{\lambda}_i, \boldsymbol{V}_i, \boldsymbol{d}_i, \boldsymbol{d}_i^{(V)}, \\ \boldsymbol{q}_i, \boldsymbol{\alpha}_i, \boldsymbol{r}_i, \boldsymbol{\nu}_i, \boldsymbol{\gamma}_i}} \sum_{j\in\mathcal{J}} \Bigg(\sum_{h\in\overline{\mathcal{H}}(j)} r_{i,h} - \sum_{h\in\mathcal{H}(j)} \Theta\big(d_{i,h}\big) \Bigg) \tag{13}$$
subject to:

Constraints (5)–(7) (14)

$$L_{i,h} - d_{i,h} \leq \alpha_{i,h} \leq L_{i,h} - d_{i,h} + M^{(1)}\big(1-\nu_{i,h}\big), \quad \forall\, h \in \overline{\mathcal{H}}(j),\ j \in \mathcal{J}; \tag{15}$$
$$0 \leq \alpha_{i,h} \leq M^{(1)}\nu_{i,h}, \quad \forall\, h \in \overline{\mathcal{H}}(j),\ j \in \mathcal{J}; \tag{16}$$
$$\nu_{i,h} \in \{0,1\}, \quad \forall\, h \in \overline{\mathcal{H}}(j),\ j \in \mathcal{J}; \tag{17}$$
$$r_{i,h} \leq q_{i,j}\pi_h + M^{(2)}\big(1-\gamma_{i,h}\big), \quad \forall\, h \in \overline{\mathcal{H}}(j),\ j \in \mathcal{J}; \tag{18}$$
$$r_{i,h} \leq \alpha_{i,h}\pi_h + M^{(2)}\big(1-\gamma_{i,h}\big), \quad \forall\, h \in \overline{\mathcal{H}}(j),\ j \in \mathcal{J}; \tag{19}$$
$$r_{i,h} \leq q_{i,j}\lambda_{i,j} + M^{(2)}\gamma_{i,h}, \quad \forall\, h \in \overline{\mathcal{H}}(j),\ j \in \mathcal{J}; \tag{20}$$
$$r_{i,h} \leq \alpha_{i,h}\lambda_{i,j} + M^{(2)}\gamma_{i,h}, \quad \forall\, h \in \overline{\mathcal{H}}(j),\ j \in \mathcal{J}; \tag{21}$$
$$\gamma_{i,h} \in \{0,1\}, \quad \forall\, h \in \overline{\mathcal{H}}(j),\ j \in \mathcal{J}; \tag{22}$$
$$q_{i,j} \in \Upsilon\big(\lambda_{i,j}, V_{i,j}\big), \quad \forall\, j \in \mathcal{J}, \tag{23}$$

with $\Upsilon\big(\lambda_{i,j}, V_{i,j}\big)$ the KKT optimality conditions of the lower-level problem (8)–(12). Problem (13)–(23) is mixed-integer programming problem with bilinear terms in (20) and (21), thus tractable with current state-of-the-art optimization algorithms and suitable for direct implementation on commercial solvers.

IV. Strategic Equilibrium in the SP

In the economic literature, the concept of equilibrium plays a key role in analyzing and estimating the outcomes of market design and is a foundational tool for exploring the interaction of competitors (e.g., consumers, in the context of this work) in a competitive environment. In this work, we make use of equilibrium techniques, in particular, the concept of Nash Equilibrium [24], to analyze the SP initiative. Formally, for a given consumer $i \in \mathcal{I}$, let $\phi(\cdot)$ be the so-called *payoff function* defined as

$$\phi\big(\boldsymbol{\lambda}_i, \boldsymbol{V}_i; \boldsymbol{\lambda}_{-i}, \boldsymbol{V}_{-i}\big) =$$
$$\max_{\substack{\boldsymbol{d}_i, \boldsymbol{d}_i^{(V)}, \\ \boldsymbol{q}_i}} \sum_{j\in\mathcal{J}} \Bigg(\sum_{h\in\overline{\mathcal{H}}(j)} R\Big(d_{i,h}, q_{i,j}, \lambda_{i,j}, \pi_h\Big) - \sum_{h\in\mathcal{H}(j)} \Theta\big(d_{i,h}\big) \Bigg)$$
subject to:

Constraints (5)–(6) and (8)–(12),

where the notation $\boldsymbol{\lambda}_{-i}$ and $\boldsymbol{V}_{-i}$ indicates the price and volume offers for all competitors, but the consumer $i \in \mathcal{I}$. Then, a *market state*, i.e., a collection of bids from all consumers, $\big((\boldsymbol{\lambda}_1^*, \boldsymbol{V}_1^*), \ldots, (\boldsymbol{\lambda}_n^*, \boldsymbol{V}_n^*)\big)$ of the SP is referred to as a *Nash Equilibrium* if the following condition holds [39]:

$$\phi\big(\boldsymbol{\lambda}_i^*, \boldsymbol{V}_i^*; \boldsymbol{\lambda}_{-i}^*, \boldsymbol{V}_{-i}^*\big) \geq \phi\big(\boldsymbol{\lambda}_i, \boldsymbol{V}_i; \boldsymbol{\lambda}_{-i}^*, \boldsymbol{V}_{-i}^*\big), \quad \forall\, \big(\boldsymbol{\lambda}_i, \boldsymbol{V}_i\big) \in \mathbb{R}_+ \times \big[\underline{V}_i, \overline{V}_i\big],\ i \in \mathcal{I}. \tag{24}$$

Identifying a Nash Equilibrium $\big((\boldsymbol{\lambda}_1^*, \boldsymbol{V}_1^*), \ldots, (\boldsymbol{\lambda}_n^*, \boldsymbol{V}_n^*)\big)$ is known to be challenging in general. Several methods and algorithmic techniques were studied in the technical literature in the past decades to tackle this challenge, such as re-writing condition (24) as an Equilibrium Problem with Equilibrium Constraints [40], and constructing iterative procedures to search for the equilibrium point (24) through the Nikaido-Isoda function [41] or Column-and-Constraint Generation [25]. In this work, we make use of an alternative and widely used approach that leverages the standard fixed-point theory [42], [43]. Roughly speaking, in the context of this work, the fixed-point-based algorithmic procedure essentially iterates across the consumers, computing their optimal *payoff functions* $\phi(\cdot)$ for a given candidate state for Nash Equilibrium, and updating such state if a better solution is found. Then, the procedure terminates when a complete iteration through all consumers results in no further updates to the candidate state. The fixed-point-based procedure is summarized in **Algorithm 1**.

Algorithm 1 Fixed-Point Algorithm - Nash Equilibrium

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1: Input: A Candidate Equilibrium ((λ*_1, V*_1), ..., (λ*_n, V*_n)).
2: while ∃ i ∈ I | φ(λ*_i, V*_i; λ*_{-i}, V*_{-i}) < φ(λ̂_i, V̂_i; λ*_{-i}, V*_{-i}) do
3:     Set {(λ*_i, V*_i)}_{i∈I} ← {(λ̂_i, V̂_i)}_{i∈I}
4:     for i ∈ I do
           Solve (λ̂_i, V̂_i) ∈ argmax_{λ_{i,j} ∈ R_+, V_{i,j} ∈ [V̲_i, V̄_i]} φ(λ_i, V_i; λ*_{-i}, V*_{-i})
           using the the optimization problem (13)–(23).
5:     end for
6: end while
7: Output: A Fixed-Point ((λ*_1, V*_1), ..., (λ*_n, V*_n)).
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In the next section, we empirically explore the bidding model and equilibrium procedure presented in this work through a set of numerical experiments using realistic data from the Brazilian power system and market environment, aiming at providing insights into the Structural Program initiative.

V. Numerical Experiments

In this section, we present a case study based on real data from the Brazilian power system and market environment. On `November 12, 2024`, the Brazilian system operator dispatched nearly 300 MW in demand response bids, making the highest recorded utilization of this resource in the country's history [44]. Our experiment is thus based on the system conditions observed on that day. For expository purposes, we focus on a single weekday, the `November 12, 2024`, i.e., $|\mathcal{J}| = 1$. The auctioned product for this day had a block duration of four hours, spanning from 19:00 to 22:00, i.e., $\overline{\mathcal{H}}(1) \triangleq \{19, 20, 21, 22\} \subset \{1, \ldots, 24\} \triangleq \mathcal{H}(1)$. Furthermore, our analysis is limited to the bids from 10 consumers ($\mathcal{I} = \{1, \ldots, 10\}$) actually dispatched in the *Structural Program* on that day[4]. As a base case, we assume that the opportunity cost $\Theta(\cdot)$ is linear in the consumption, i.e., $\Theta\big(d_{i,h}\big) = \sum_{h\in\mathcal{H}(j)} \theta_i^{(\mathrm{L})} d_{i,h},\ \forall\ i \in \mathcal{I}$, with $\theta_i^{(\mathrm{L})}$ an *a-priori* defined parameter. Column 2 of Table IV in Appendix A displays the values of $\theta_i^{(\mathrm{L})}$ considered in the base case for each consumer $i \in \mathcal{I}$. Furthermore, Table III in Appendix A presents the lower- and upper-bounds on the volume bid for each consumer $i \in \mathcal{I}$.

In accordance with the current day-ahead operational planning guidelines in Brazil, non-conventional renewable generation

[4] The remaining amount was dispatched through the *Availability Program*, which is not considered in this case study.

sources (e.g., wind, solar, and small-hydro power plants) are treated as negative loads. Thus, the total demand considered in this case study corresponds to a system's net load (see the black curve in Fig. 3 in Appendix A). The thermoelectric generation fleet consists of the 45 power plants dispatched on `November 12, 2024`, with a total available capacity of approximately 12 GW and variable operating costs reaching up to 1,374.12 R$/MWh. Fig. 1 in Appendix A illustrates the supply curve for the generation mix used in the case study. For energy spot prices, we consider the real values reported by the Brazilian market operator (see the green curve in Fig. 3 in Appendix A). The following section details the key results and discusses the main insights found in this numerical experiment.

A. Operative Results, Optimal Bidding and Market Equilibrium

As a general operative result at equilibrium, Fig. 2 in Appendix A provides an overview of the total dispatch of available system resources, focusing on the time block during which the demand response product is auctioned (from 19:00 to 22:00). Furthermore, Fig. 3 in Appendix A presents the system's marginal costs, comparing operating conditions with (blue curve) and without (orange curve) demand response in the resource mix. Note that when demand response is excluded, the marginal costs rise to the value of the most expensive thermoelectric generator (1,374.12 R$/MWh), whereas, with demand response, the highest marginal cost is reduced to approximately 1,200.00 R$/MWh with an average value of 1,066.87 R$/MWh during the demand response product's hours. We highlight that since the Brazilian market operator uses different (simplified) premises for the system to calculate the energy spot price, it reaches a lower (maximum) price of 965.37 R$/MWh. We argue that incorporating DR into the resource mix at equilibrium enhances system flexibility, leading to a reduction in overall marginal costs. Additionally, we emphasize that it should be factored into the pipeline for electricity pricing to better reflect the resulting increase in operational flexibility, potentially contributing to lower energy spot prices.

With respect to the strategic and equilibrium behavior of the consumers, we focus on fixed-point Nash Equilibrium found by running **Algorithm 1** (hereinafter referred to as *Equilibrium*) and the optimal bidding of each consumer $i \in \mathcal{I}$ assuming that the competitors are bidding their maximum reduction capacity, i.e., $\overline{V}$ (hereinafter referred to as *Individual*). Furthermore, to enhance the bidding and market equilibria analysis, we also consider a quadratic opportunity cost functional for each consumer $i \in \mathcal{I}$, i.e., $\Theta\big(d_{i,h}\big) = \theta_i^{(\mathrm{Q})}\big(\sum_{h\in\mathcal{H}(j)} d_{i,h}\big)^2, \forall\ i \in \mathcal{I}$, with $\theta_i^{(\mathrm{Q})}$ an *a-priori* specified parameter. Column 3 of Table IV in Appendix A displays the values of $\theta_i^{(\mathrm{Q})}$ for each consumer $i \in \mathcal{I}$. Table I and Table II present the price and volume bids[5] of each consumer for each context analyzed (*Equilibrium* and *Individual*) for the linear and quadratic opportunity cost, respectively.

Firstly, for linear opportunity costs, all price bids converged at 1,006.11 R$/MWh. Upon analyzing the system's marginal costs, we observe that this price level corresponds to the average marginal cost over the time block during which the demand response product is auctioned (from 19:00 to 22:00). Since the SP clearing process resembles a pay-as-bid mechanism, participants aim to match this threshold price in order to fully dispatch their volume bids and maximize their revenue in the auction. As a result, the alignment of price bids with the average marginal cost during the product block duration stems from the inflexible scheduling (same dispatch for all hours) of demand response bids. On the other hand, the optimal volume bid corresponds to the maximum reduction capacity of each consumer for linear opportunity costs (see Column 3 of Table I). However, with a quadratic opportunity cost function, consumers #2, #5, and #8 withhold part of their volume capacity due to the high penalties associated with larger reduction levels (see Column 3 of Table II). As a result, under quadratic opportunity costs, price bids converge at higher levels (1,016.05 R$/MWh). This occurs because the reduced overall availability of demand response raises the system's average marginal cost during the auction period (19:00–22:00). Consequently, consumers adjust their strategic bids to capitalize on this condition, increasing their price bids to maximize revenue. Finally, we observe that the strategic behavior of the consumers in the *Individual* context is to bid a price at the system marginal cost level (1,006.11 R$/MWh) but adjust the volume bid to match the optimal equilibrium behavior according to the opportunity cost function topology.

[5]Fig. 4 and Fig. 5 in Appendix A displays the price and volume evolution at each iteration of **Algorithm 1**.

TABLE I
OPTIMAL PRICE AND VOLUME BID FOR ALL CONSUMERS $i \in \mathcal{I}$ ASSUMING A LINEAR OPPORTUNITY COST FOR BOTH INDIVIDUAL AND EQUILIBRIUM CONTEXTS.

Consumer	Individual		Equilibrium	
	Price Bid (R$/MWh)	Volume Bid (MW)	Price Bid (R$/MWh)	Volume Bid (MW)
1	1006.11	10.0	1006.11	10.0
2	1006.11	38.0	1006.11	38.0
3	1006.11	5.0	1006.11	5.0
4	1006.11	5.0	1006.11	5.0
5	1006.11	50.0	1006.11	50.0
6	1006.11	11.0	1006.11	11.0
7	1006.11	6.0	1006.11	6.0
8	1006.11	33.0	1006.11	33.0
9	1006.11	17.0	1006.11	17.0
10	1006.10	27.0	1006.11	27.0

TABLE II
OPTIMAL PRICE AND VOLUME BID FOR ALL CONSUMERS $i \in \mathcal{I}$ ASSUMING A QUADRATIC OPPORTUNITY COST FOR BOTH INDIVIDUAL AND EQUILIBRIUM CONTEXTS.

Consumer	Individual		Equilibrium	
	Price Bid (R$/MWh)	Volume Bid (MW)	Price Bid (R$/MWh)	Volume Bid (MW)
1	1006.11	10.0	1016.05	10.0
2	1006.11	12.6	1016.05	12.7
3	1006.11	5.0	1016.05	5.0
4	1006.11	5.0	1016.05	5.0
5	1006.11	13.2	1016.05	13.4
6	1006.11	11.0	1016.05	11.0
7	1006.11	6.0	1016.05	6.0
8	1006.11	16.8	1016.05	16.9
9	1006.11	17.0	1016.05	17.0
10	1006.10	27.0	1016.05	27.0

VI. CONCLUSION

This paper provides a comprehensive evaluation of the Structural Program (SP) for demand response (DR) in Brazil, analyzing its potential to enhance system flexibility and improve market efficiency. We first model the consumer bidding strategy in the SP and then design a procedure to evaluate a Nash Equilibrium through a fixed-point algorithm. Our findings demonstrate that optimized bidding strategies can significantly improve participant outcomes, increase system flexibility, and optimize the functioning of DR programs. Through a case study based on real data of the Brazilian power system and market, we observed that, at equilibrium, the optimal price bid aligns with the average system marginal operational cost over the time block during which the demand response product is auctioned. On the other hand, the optimal volume bid, however, is influenced by the consumer's opportunity cost function. Future work might explore a more detailed clearing to be incorporated into the model and consider uncertainties in the decision-making process.

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Appendix A

This appendix provides a detailed overview of the data used in the numerical experiment, along with the convergence results obtained from running **Algorithm 1** in the case study outlined in Section V. Fig 1 shows the thermoelectric generation mix supply curve used in the case study, which consists of the 45 power plants dispatched on `November 12, 2024`, with a total installed capacity of approximately 12 GW and variable operating costs up to 1,374.12 R$/MWh. Additionally, Table III provides the lower $(\underline{V})$ and upper $(\overline{V})$ bounds on the volume bid for each consumer $i \in \mathcal{I}$ assumed in the experiment and Table IV lists the coefficients for the linear $(\theta^{(L)})$ and quadratic $(\theta^{(Q)})$ opportunity cost functions considered for each consumer $i \in \mathcal{I}$ in the case study.

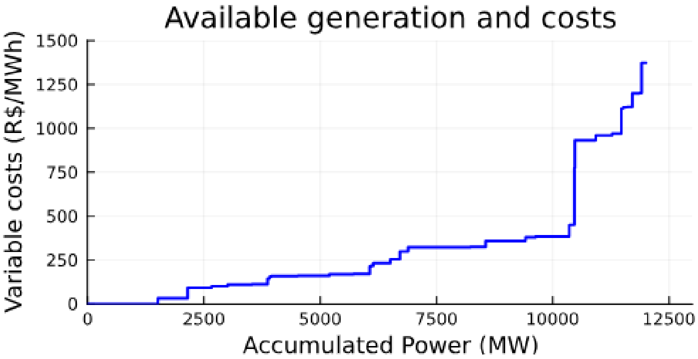

Fig. 1. Thermoelectric generation mix supply curve composed of the 45 power plants that were dispatched on `November 12, 2024`, with a total installed capacity of approximately 12 GW and variable operating costs reaching up to 1,374.12 R$/MWh.

TABLE III
Lower- $(\underline{V})$ and upper-bounds $(\overline{V})$ on the volume bid for each consumer $i \in \mathcal{I}$.

Consumer	Lower-Bound $(\underline{V})$ (MW)	Upper-Bound $(\overline{V})$ (MW)
1	0	10
2	0	38
3	0	5
4	0	5
5	0	50
6	0	11
7	0	6
8	0	33
9	0	17
10	0	27

TABLE IV
Coefficients of the Linear $(\theta^{(L)})$ and Quadratic $(\theta^{(Q)})$ Opportunity Cost Functions for each Consumer $i \in \mathcal{I}$.

Consumer	Linear Coefficients $(\theta^{(L)})$	Quadratic Coefficients $(\theta^{(Q)})$
1	500.0	10.0
2	500.0	10.0
3	500.0	10.0
4	480.0	9.6
5	475.0	9.5
6	475.0	9.5
7	425.0	8.5
8	375.0	7.5
9	340.0	6.8
10	215.0	4.3

Regarding the operative results, Fig. 2 presents an overview of the total dispatch of available system resources, focusing on the time block during which the demand response product is auctioned and Fig 3 displays the system's marginal costs, considering operating conditions with and without demand response in the resource mix, energy spot prices, and total net load.

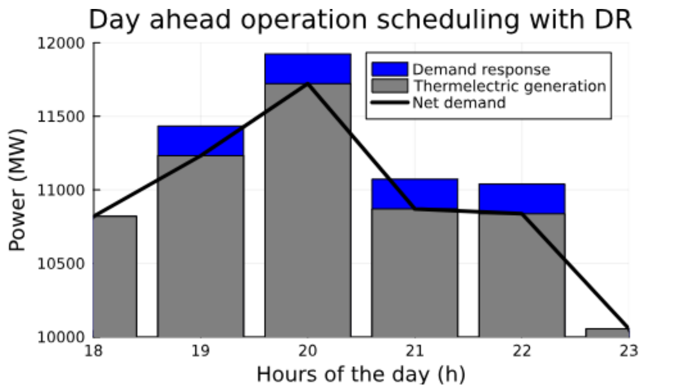

Fig. 2. Overview of the total dispatch of available system resources, focusing on the time block during which the demand response product is auctioned.

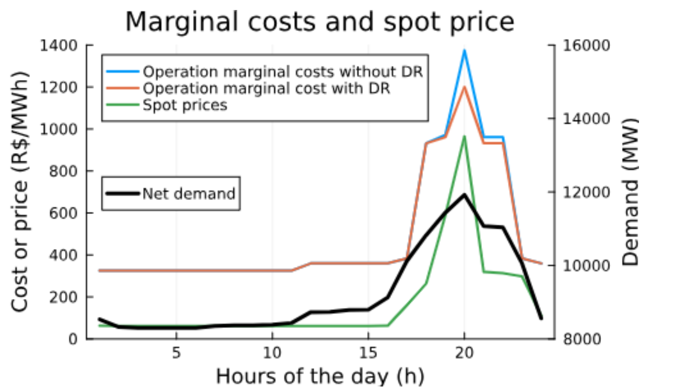

Fig. 3. System's marginal costs, considering operating conditions with (blue curve) and without (orange curve) demand response in the resource mix, energy spot prices (green curve), and total net load (black curve).

Finally, with respect to the convergence of **Algorithm 1**, Fig. 4 illustrates the fixed-point progression across iterations in the context of a linear opportunity cost function, where initial volume bids correspond to the maximum reduction capacity of each consumer $i \in \mathcal{I}$ (Column 3 of Table III) and price bids reflect those actually submitted to the market on `November 12, 2024`. Similarly, Fig. 5 presents the evolution of the fixed-point algorithm in the case of a quadratic opportunity cost function.

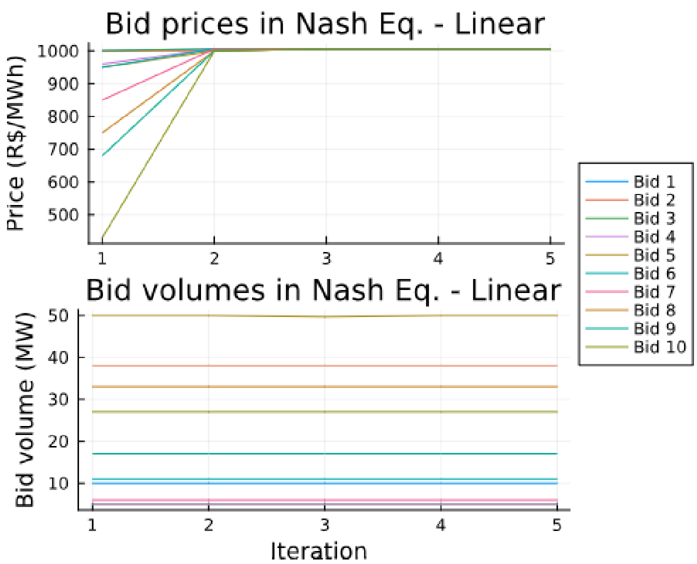

Fig. 4. Fixed-point price and volume bids progression across iterations of **Algorithm 1** in the context of a linear opportunity cost function.

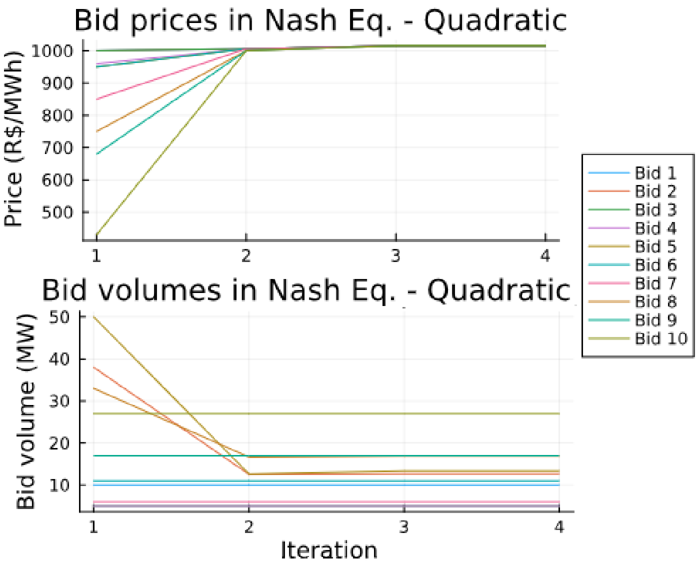

Fig. 5. Fixed-point price and volume bids progression across iterations of **Algorithm 1** in the context of a quadratic opportunity cost function.