ERIKSSON: a global market model for green hydrogen and its derivatives with strategic behavior and detailed supply

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***Abstract*—In response to global goals for reducing greenhouse gas emissions, a market for green hydrogen and hydrogen derivatives produced from renewable energy sources is currently emerging. The structure of the emerging market is unclear due to a lack of experience with such a market and high uncertainties regarding future costs and demand. We want to learn how the market could develop to gain insights for policymakers and decision-makers. To this end, we introduce the new global trade model, ERIKSSON, simulating the future global market for green hydrogen and hydrogen derivatives. The model includes specific characteristics of such a market, like strategic behavior, a detailed mapping of global renewable energy potentials, conversion and competition between multiple commodities, temporal availability of renewable energy production, and losses. ERIKSSON builds a basis for further research on possible market equilibria.**

I. Introduction

Due to global ambitions to reduce greenhouse gas emissions, a market for green hydrogen and hydrogen derivatives produced from renewable energy sources (RES) is currently in the start-up phase. Knowledge about possible market development and equilibria is important for policy- and decision-makers to set up an efficient strategy for cost-efficient and resilient supply and corresponding infrastructure investments. Countries like Germany expect future demand for hydrogen and hydrogen derivatives and have started to set up an import strategy. For hydrogen, there is a debate in science and politics on how much imports will be required in the future and to which degree these imports will be supplied by pipeline or ship. The level of demand depends on the long-term price development, which is uncertain from today's perspective.

The question of import quantity and method is complex due to a lack of experience, as a market for hydrogen and hydrogen derivatives has not yet existed on a large scale. Markets for hydrogen and hydrogen derivatives might have similarities and differences to existing markets for energy commodity markets like natural gas and oil. First, the RES potential is distributed globally instead of concentrated in a few countries. However, the RES potentials with the lowest production costs are concentrated in a few countries, similar to oil and gas resources. Thus, the global market to develop might be an oligopolistic market. Second, due to the volatility of RES, green hydrogen and hydrogen derivatives production is time-variant instead of constant. Third, transport and storage costs are considerably higher for hydrogen than for natural gas. For hydrogen derivatives, these costs are similar to oil. Storage is of particular importance for market development because it balances out volatile RES production. Fourth, the markets for hydrogen and derivatives are connected. On the one hand, the production of green hydrogen and hydrogen derivatives use the same RES potentials. On the other hand, hydrogen and hydrogen derivatives can be converted into each other. Fifth, the transport and conversion of hydrogen and its derivatives might be subject to higher losses than natural gas or oil.

A large stream of literature investigates specific aspects of the future markets for hydrogen and hydrogen derivatives. [1] et al. estimate global supply costs and production potentials for green hydrogen from dedicated ress. Moritz et al. [2] expand this work by hydrogen derivatives and argue that transportation costs dominate the supply costs of hydrogen, and production costs dominate the supply costs of hydrogen derivatives. Therefore, regional, pipeline-based hydrogen markets and global, shipping-based derivative markets might emerge. Genge et al. [3] review 30 studies on supply costs and find a five-fold difference in cost projections for 2050, which stresses the high uncertainty regarding cost developments.

While analyzing individual aspects is insightful, market modeling enables understanding future markets in greater depth. A few works exist that model markets for hydrogen and hydrogen derivatives with different scopes. Guo et al. [4] investigate a regional market in northern China. A number of studies [5–11] examine a global market with varying degrees of spatial resolution. Some of these studies [7, 8, 10, 11] use world regions constituting one or multiple countries as spatial structures. Within these studies, the degree of detail spans from six world regions [10] to 37 world regions [7]. Some studies [5, 6, 9] model the world on a country level but do not include all countries of the world. Antweiler and Schlund [9] consider the 48 most relevant countries, Ortmann et al. [7] do not cover most of Africa, the Middle East, and Central Asia. Schönfisch [5] covers most of the world, with the least coverage in Africa, and Ikonnikova et al. [6] cover almost the entire world.

While most of the existing models depict a single-commodity hydrogen market, some incorporate multiple commodities. Barner [11] models green hydrogen and ammonia markets. Zhang et al. [8] incorporate a hydrogen market into a global multi-commodity energy system model including grey, blue, pink, and green hydrogen. Schönfisch [5] proposes an integrated natural gas and low-carbon hydrogen market model.

Some of the models include the strategic behavior of market participants. Antweiler and Schlund [9] propose a model for an emerging global hydrogen market characterized by long-term contract (LTC)s. Their hydrogen trade under LTCs relies on projects that can be introduced in each time period, where they compete to enter a Nash-Cournot equilibrium. However, instead of competing based on quantity, the competition is centered around gaining market access, as the capacity is always fully utilized. Barner [11], Dupke et al. [10], and Schönfisch [5] propose mixed complementarity problem (MCP) models where traders strategically choose their trade volumes in a Cournot competition. However, the case study presented by Schönfisch [5] only depicts perfect competition. Guo et al. [4] model strategic behavior of individual participants regarding the uncertain availability of RES with a game model as mixed integer linear programm (MILP), but not as Cournot competition. The rest of the studies assume either explicitly perfect competition or implicitly minimizing total costs from a central planner's perspective.

Barner [11] and Dupke et al. [10] incorporate price-elastic demand. All other studies assume inelastic demand. Two studies depict the seasonality of supply and demand. Barner [11] models two seasons, representing summer and winter. Guo et al. [4] model in an hourly time-resolution.

Existing studies model the supply side in various degrees of detail. Antweiler and Schlund [9] use data from Brändle et al. [1] to derive country-specific supply curves for RES-based hydrogen. Schönfisch [5] also uses data from Brändle et al. and models the RES supply by up to nine RES classes per country and distinguishes three RES types (photovoltaic (PV), wind offshore, and wind offshore) and their production costs and potentials for different annual capacity factors. Ortmann et al. [7] use a similar approach based on data from Moritz et al. [2]. Barner [11] conducts a GIS analysis to estimate RES potentials and constructs piecewise linear supply curves for each world region. Dupke et al. [10] use electrolysis capacities announced for 2030 as production potentials and calculate production costs based on a continent-specific electricity price. Guo et al. [4] assume convex quadratic supply curves for each producer node. Ikonnikova et al. [6] use country-specific supply cost functions for grid-based electricity. Zhang et al. [8] model grid-based and dedicated RES-based hydrogen production based on another model [12].

We classify features of the existing models in Table I. While these existing market modeling publications provide valuable insights, market equilibria, and trade flows in future hydrogen and hydrogen derivatives markets remain open to further exploration.

We contribute a new model named ERIKSSON, which simulates the global market for green hydrogen and hydrogen derivatives. The model includes strategic behavior, a detailed mapping of global RES potentials, conversion and competition between multiple commodities, temporal availability of RES production, and losses. ERIKSSON is inspired by the global gas market model COLUMBUS [13, 14].

II. Model description

ERIKSSON is a multi-commodity partial equilibrium model for the global green hydrogen and derivatives markets. We model the global markets as MCP in which different players aim to optimize their profit. ERIKSSON includes eight types of players: producers, exporters, pipeline operators, ship operators, export terminal operators, import terminal operators, converters, and storage operators. ERIKSSON's temporal structure consists of months $m \in M$ and years. The current version spans one year. The spatial structure is based on the graph theory and consists of nodes $n \in N$ and a set of vertices $V \subset (N \times N)$. Consumption and demand nodes act as sinks, since they have demand, and as sources, since they have RES potentials. Moreover, we introduce harbor nodes $h \in H \subset N$, where terminal operators load and unload ships for maritime transport. Vertices represent transport routes between nodes. There are different sets for vertices representing shipping routs $V^{ship} \subset V$ and pipeline routes $V^{pipeline} \subset V$. Figure 2 shows a map of the model's spatial structure. Exporters sell, producers produce, infrastructure operators transport, and storage operators store the following green energy commodities $c \in C$: hydrogen, methane, ammonia, methanol, Fischer-Tropsch-fuels, and LOHC. Producers can access different classes of RES potentials $r \in R$ and invest in production plants with two production profiles $p \in P$: volatile and baseload. Each producer is vertically integrated with a dedicated exporter to which it sells via a long-term contract with 100 % take-or-pay rate. The trade is modelled as Cournot competition on a global spot market, in which oligopolistic exporters $e \in E$ strategically choose sales volumes to exert market power, influence prices, and maximize their profit. Each exporter is active at its nodes $N^{\mathrm{e}} \in N$. Exporters can decide between two transport methods $t \in T$: ship and pipeline. Converters can convert commodities into each other and are modelled as different converter types $(c \times c1) \in CT$. We incorporate all converters from hydrogen to derivatives as well as Ammonia Cracking and LOHC Dehydrogenation. Storage operators can store in two storage types $s \in S$: tank and cavern storage. The cavern storage potential is constrained and not available in all nodes. Table II gives an overview of all sets, variables, and parameters of ERIKSSON.

As this paper focuses on the mathematical model description, we only briefly address input data. Most data is based on the EWI Global PtX Cost tool [15], which is an evolution of the work by Brändle et al. and Moritz et al. [1, 2]. We use a two-step modeling with a pre-optimization approach to shift some of the computational burdens away from ERIKSSON. By this, we can depict RES production in more detail as if all was modeled in a single model. In the pre-optimization, we use a linear invest and dispatch optimization model to calculate the production costs of integrated production plants for hydrogen or hydrogen derivatives. The model designs plants for either volatile or baseload production profiles. For the volatile profile, the production costs are minimized to produce an annual quantity. Production costs for the baseload profile are minimized to cover an hourly constant production using storage tanks. The linear model allows the calculation of production costs in greater detail, with hourly temporal resolution. The production costs optimization is performed for 119 countries, each with up to nine RES classes. RES classes are distinguished by RES type (wind onshore, wind offshore, and PV) and the average capacity factor (wind onshore and PV) or the water depth (wind offshore). Each RES class has an individual time series of hourly capacity factors and the corresponding technical potential. We use the results from the invest and dispatch model as input for ERIKSSON. Inputs include the levelized costs of production, the annual production potential, the temporal availability of production, and the losses for each transport route.

The following subsections describe the optimization problems of all players and the market clearing conditions.

A. The producer's problem

Each producer at a node n operates the production of one or multiple commodities c. We assume that each producer is vertically integrated with an exporter e. Thus, the exporter's decision to sell determines the producer's annual production quantity $prod_{n,c,r,p}$. Only one producer and one exporter can be active at each node. While a producer is only active at a single node, an exporter can control the production in multiple nodes. The producer earns revenue by selling to the exporter at the supply costs $\lambda_{m,e,n,c}$ via a long-term contract with a 100 % take-or-pay rate.[1] The supply costs $\lambda_{m,e,n,c}$ represents the marginal costs of one additional unit of supply at node n for the exporter e. In return, the producer must cover the production costs $\mathrm{cost}^{\text{production}}_{n,c,r,p}$. The producer can produce from multiple RES classes r at different production costs and can choose from two delivery types p. The producer aims to maximize its profit $\Pi^{\text{production}}_{n,e}$ by producing at the lowest possible costs (Equation 1). The monthly availability $\mathrm{avail}_{m,n,c,r,p}$ translates the annual production into monthly production quantities based on the RES classes' seasonality for each m. The production quantity is subject to the RES potential constraint (Equation 2). In this constraint, the required electricity production for the commodity production $prod_{n,c,r,p}$ is calculated based on the production efficiency $\mathrm{eff}^{production}_{n,c,r,p}$. The production efficiency includes energy efficiencies of all conversion processes and curtailment. For

[1] With this approach, levelized costs reflect production costs. The development of production capacities is bound to high upfront investments. Assuming a similar risk allocation as in other energy commodity markets, upfront investments are secured by long-term contracts.

every RES class, the required electricity production can not exceed the RES potential $\text{pot}_{n,r}^{production}$. The dual variable $\alpha_{n,r}$ represents the RES potential scarcity rent or marginal benefit of one additional unit of RES potential to the producer.

$$\max_{prod_{n,c,r,p}} \Pi_{e,n,c}^{\text{production}}(prod_{n,c,r,p}) = \sum_{m,r,p} \Big(prod_{n,c,r,p} \cdot \text{avail}_{m,n,c,r,p} \cdot (\lambda_{m,e,n,c} - \text{cost}_{n,c,r,p}^{\text{production}}) \Big) \quad \forall n \in N^e, c. \tag{1}$$

s.t.

$$\text{pot}_{n,r}^{production} - \sum_{m,c,p} \frac{prod_{n,c,r,p} \cdot \text{avail}_{m,n,c,r,p}}{\text{eff}_{n,c,r,p}^{production}} \geq 0 \quad (\alpha_{n,r}) \quad \forall n, r. \tag{2}$$

B. The exporter's problem

The profit maximization problem of each exporter e is defined by Equation 3. Each exporter aims to maximize its profit Π_e^{exporter} by strategically choosing sales volumes $sell_{m,e,n,c}$ of commodities c to nodes n in each month m on a global spot market. For each sale conducted at a node n, the exporter pays the supply costs $\lambda_{m,e,n,c}$ and gets the domestic market clearing price $\beta_{m,n,c}$ as revenue. $\lambda_{m,e,n,c}$ is the dual variable of the physical exporter balance. $\beta_{m,n,c}$ is the dual variable of the domestic market clearing balance (see section II-J). The conjectural variation $\text{cv}_{e,n} \in [0,1]$ defines the exporter's strategic behavior towards a node. If the conjectural variation equals zero, the exporter observes the domestic market clearing price. Since the exporter does not know how his decision to sell impacts the market clearing price, he behaves perfectly competitively as a price taker. If the conjectural variation equals one, the exporter knows the linear inverse demand function in the nodes he sells at. Thus, he behaves strategically as a price maker and assumes that all other exporters act similarly. The closer the conjectual variation is to one, the more strategic the exporter behaves.

$$\max_{sell_{m,e,n,c}} \Pi_e^{\text{exporter}}(sell_{m,e,n,c}) = \sum_{m,n,c} \Big[sell_{m,e,n,c} \cdot \Big(\text{cv}_{e,n} \cdot \big(\text{p}_{n,c}^{\text{ref}} - \text{slope}_{m,n,c} \cdot (\sum_{e} sell_{m,e,n,c} - buy_{m,e=\text{arbitrageur},n,c}) \big) + (1 - \text{cv}_{e,n}) \cdot \beta_{m,n,c} - \lambda_{m,e,n,c} \Big) \Big] \quad \forall e. \tag{3}$$

The exporter also aims to minimize the supply costs $C_{e,n,n1}^{\text{exporter}}$ of the physical $flow_{m,e,n,n1,c,t}$ to deliver his sales (Equation 4). The monthly physical flows are subject to the capacity constraints of the transport infrastructure, i.e., pipelines, export terminals, import terminals, and transport ships (Equation 5-6). Infrastructure and shipping capacities are decision variables of the respective infrastructure operators (see sections II-D-II-G). We assume that transport ships shuttle between origin and destination. The dual variables of the capacity constraints represent congestion rents or the marginal benefit of one extra infrastructure unit to the exporter. $\gamma_{m,n,n1,c}$ is the pipeline congestion rent between nodes n and $n1$ for commodity c in month m. $\delta_{m,c}$ is the shipping fleet congestion rent of commodity c in month m. $\eta_{m,h,c}$ and $\theta_{m,h,c}$ are the export terminal and import terminal congestion rents at harbor node h for commodity c in month m, respectively.

$$\min_{flow_{m,e,n,n1,c,t}} C_{e,n,n1}^{\text{exporter}}(flow_{m,e,n,n1,c,t}) = \sum_{m,(n \times n1) \in V,c,t} \Big(flow_{m,e,n,n1,c,t} \cdot \Big[\Big(\text{vcost}_{n,n1,c,t}^{\text{transport}} + \text{vcost}_{n,c,t}^{\text{export}} + \text{vcost}_{n1,c,t}^{\text{import}} \big) - \big(\lambda_{m,e,n1,c} - \lambda_{m,e,n,c} \big) \Big] \Big) \quad \forall e. \tag{4}$$

s.t.

$$cap_{n,n1,c}^{\text{pipeline}} - \sum_{e,t=pipeline} flow_{m,e,n,n1,c,t} \geq 0 \quad (\gamma_{m,n,n1,c}) \quad \forall m, (n \times n1) \in V, c. \tag{5}$$

$$cap_c^{\text{ship}} \cdot \frac{8760}{12} \cdot \text{speed} - \sum_{e,(h \times h1) \in V^{shipping},t=ship} \Big(2 \cdot flow_{m,e,h,h1,c,t} \cdot \text{dist}_{h,h1}^{\text{ship}} \Big) \geq 0 \quad (\delta_{m,c}) \quad \forall m, c. \tag{6}$$

$$cap_{h,c}^{\text{exp}} - \sum_{e,h1,t=ship} flow_{m,e,h,h1,c,t} \geq 0 \quad (\eta_{m,h,c}) \quad \forall m, h, c. \tag{7}$$

$$cap_{h,c}^{\text{imp}} - \sum_{e,h1,t=ship} flow_{m,e,h1,h,c,t=ship} \geq 0 \quad (\theta_{m,h,c}) \quad \forall m, h, c. \tag{8}$$

C. The buyer's problem

Next to exporters with dedicated producers, we also model an exporter that performs arbitrage trading. This arbitrageur can sell globally and has no dedicated producers but a dedicated buyer, which can buy globally at market prices. The buyer's objective at each node n in each month m and for each commodity c is to maximize profit by buying the quantity $buy_{m,e,n,c}$ as cheap as possible at price $\beta_{m,n,c}$ and selling to the arbitrageur at the arbitrageur's supply cost $\lambda_{m,e,n,c}$. Note that the buyer cannot make profit from this operation, but the buyer and the arbitrageur in combination.

$$\max_{buy_{m,e,n,c}} \Pi_e^{\text{buy}}(buy_{m,e,n,c}) = \sum_{m,n,c} \big(buy_{m,e,n,c} \cdot (\lambda_{m,e,n,c} - \beta_{m,n,c}) \big) \quad e = \text{arbitrageur} \tag{9}$$

D. The pipeline operator's problem

Each pipeline between two nodes $(n, n1)$ is operated by one pipeline operator who aims to maximize his profit. We assume that pipeline operators are subject to regulation and get paid their variable costs by exporters. Therefore, pipeline operators only make profit from the congestion rent $\gamma_{m,n,n1,c}$, which is also paid by the exporter. The pipeline operator can invest in pipeline capacity $cap_{n,n1,c}^{\text{pipeline}}$. Annual costs from investments are calculated based on the pipeline capacity $cap_{n,n1,c}^{\text{pipeline}}$, specific investment costs sic_c^{pipeline}, the annuity a_c^{pipeline}, as well as fixed operation and maintenance (FOM) costs $\text{fom}_c^{\text{pipeline}}$. The pipeline operator's objective function is defined as follows:

$$\begin{aligned}
&\max_{cap^{\text{pipeline}}_{n,n1,c}} \Pi^{\text{pipeline}}_{n,n1,c}(cap^{\text{pipeline}}_{n,n1,c}) = \\
&cap^{\text{pipeline}}_{n,n1,c} \cdot \Big(\sum_m \gamma_{m,n,n1,c} \\
&- \text{sic}_c^{\text{pipeline}} \cdot \text{dist}_{n,n1}^{pipeline} \cdot (\text{a}_c^{\text{pipeline}} + \text{fom}_c^{\text{pipeline}}) \Big) \\
&\quad \forall (n \times n1) \in V^{\text{pipeline}}, c.
\end{aligned} \tag{10}$$

E. The ship operator's problem

We assume that one ship operator runs the fleet of transport ships for each commodity c. Because the global freight shipping market is highly competitive, ship operators behave perfectly competitively. The ship operators aim to maximize their profit by investing in additional shipping capacity until the marginal investment costs equal the marginal benefits. The marginal benefits are shipping congestion rent $\delta_{m,c}$, which is the dual variable of the ship capacity constraint (Equation 6). Equation 11 shows the objective function of the ship operator:

$$\begin{aligned}
&\max_{cap^{\text{ship}}_{c}} \Pi^{\text{ship}}_{c}(cap^{\text{ship}}_{c}) = \\
&cap^{\text{ship}}_{c} \cdot \Big(\sum_m \delta_{m,c} - \text{sic}_c^{\text{ship}} \cdot (\text{a}_c^{\text{ship}} + \text{fom}_c^{\text{ship}}) \Big) \quad \forall c.
\end{aligned} \tag{11}$$

F. The export terminal operator's problem

An export terminal operator is active at each harbor node $h \in H \subset N$ for each commodity c. The export terminal operator receives commodities from exporters and loads them onto ships. The export terminal includes harbor cargo facilities. In the case of hydrogen and methane, it also includes a liquefaction terminal. The commodities are then sent to import terminals by using tankers (as described in the ship operator's problem in section II-E). We assume perfect competition between export terminal operators. Thus, terminal operators can only make profit from the export terminal congestion rent $\eta_{m,h,c}$, which is determined by the export terminal capacity constraint (Equation 7). The objective function of the export terminal operator results in the following:

$$\begin{aligned}
&\max_{cap^{\text{export}}_{h,c}} \Pi^{\text{export}}_{h,c}(cap^{\text{export}}_{h,c}) = \\
&cap^{\text{export}}_{h,c} \cdot \Big(\sum_m \eta_{m,h,c} - \text{sic}_c^{\text{export}} \cdot (\text{a}_c^{\text{export}} + \text{fom}_c^{\text{export}}) \Big) \\
&\quad \forall h, c.
\end{aligned} \tag{12}$$

G. The import terminal operator's problem

An import terminal operator is active at each harbor node h for each commodity c. The import terminal operator unloads commodities from ships and transfers them to pipeline operators. The import terminal includes harbor cargo facilities and a regasification terminal in the case of hydrogen and methane. The import terminal congestion rent $\theta_{m,h,c}$ is determined by the import terminal capacity constraint (Equation 8). We assume that the import terminal operators compete in a perfectly competitive market. Thus, we define the objective function analogously to the export terminal operators:

$$\begin{aligned}
&\max_{cap^{\text{import}}_{h,c}} \Pi^{\text{import}}_{h,c}(cap^{\text{import}}_{h,c}) = \\
&cap^{\text{import}}_{h,c} \cdot \Big(\sum_m \theta_{m,h,c} - \text{sic}_c^{\text{import}} \cdot (\text{a}_c^{\text{import}} + \text{fom}_c^{\text{import}}) \Big) \\
&\quad \forall h, c.
\end{aligned} \tag{13}$$

H. The converter's problem

At every node n a converter participates in the domestic market as an arbitrageur between two commodities $(c, c1)$. To maximize profit, the converter can invest in conversion capacity $cap^{\text{conversion}}$ and decides the conversion $con_{m,n,c,c1}$ in month m. For the commodity arbitrage, converters get the commodity price of their output commodity $\beta_{m,n,c1}$ and pay the price for their input commodity $\beta_{m,n,c}$. The required amount of the input commodity c is defined by the conversion efficiency $\text{eff}_{c,c1}^{\text{conversion}}$. In addition, the converter has to pay investment costs and variable conversion costs. The resulting objective function is defined in (Equation 14). The monthly conversion $con_{m,n,c,c1}$ is restricted by the conversion capacity constraint (Equation 15). The dual variable $\mu_{m,n,c,c1}$ represents the conversion capacity scarcity rent or marginal benefit of one additional unit of conversion capacity to the converter.

$$\begin{aligned}
&\max_{cap^{\text{conversion}}_{n,c,c1},\, con_{m,n,c,c1}} \Pi^{\text{conversion}}_{n,c,c1}(cap^{\text{conversion}}_{n,c,c1},\, con_{m,n,c,c1}) = \\
&\sum_m \Big(con_{m,n,c,c1} \cdot \Big(\beta_{m,n,c1} - \frac{\beta_{m,n,c}}{\text{eff}^{\text{conversion}}_{c,c1}} \Big) \Big) \\
&- \Big(\text{sic}^{\text{conversion}}_{n,c,c1} \cdot \text{a}^{\text{conversion}}_{n,c,c1} + \text{fom}^{\text{conversion}}_{n,c,c1} \Big) \cdot cap^{\text{conversion}}_{n,c,c1} \\
&- \sum_m con_{m,n,c,c1} \cdot \text{vcost}^{\text{conversion}}_{n,c,c1} \\
&\quad \forall n, (c \times c1) \in CT.
\end{aligned} \tag{14}$$

s.t.

$$\begin{aligned}
&cap^{\text{conversion}}_{n,c,c1} - con_{m,n,c,c1} \geq 0 \quad (\mu_{m,n,c,c1}) \\
&\quad \forall m, n, (c \times c1) \in CT.
\end{aligned} \tag{15}$$

I. The storage operator's problem

At every node n and for each commodity c, a storage operator participates in the domestic market as a temporal arbitrageur, storing the commodity between different months m. The storage operator can choose between different storage types s $\in \{cavern, tank\}$. To maximize profit, the operator can invest in storage capacity $cap^{\text{storage}}_{n,c,s}$ and decides the storage depletion $out^{\text{storage}}_{m,n,c,s}$ and storage injection $in^{\text{storage}}_{m,n,c,s}$ in month m. The storage operator receives the price $\beta_{m,n,c}$ for storage depletion or pays it for injection. In addition, the storage operator has to pay investment costs for capacity deployment and variable costs for storage injection. Equation 16 shows the storage operator's objective function. Several constraints restrict the storage operator's problem. Equation 17 shows the storage balance constraint. The storage level in the next month $lvl_{m+1,n,c,s}$ is defined by the storage level, depletion, and injection in the current month. Equation 18 states that the storage capacity restricts the storage level. Equation 19 says that the storage capacity is restricted by the storage potential $\text{pot}^{\text{storage}}_{n,c,s}$. Tank potential is assumed to be unlimited. Cavern potential is limited and only available in certain nodes. Equation 20 and 21 state that the storage depletion and injection capacity constrain the monthly storage depletion and injection. We define the storage injection and depletion capacity as linear to the storage capacity, with a maximum of two storage cycles per year. For the storage depletion and storage injection constraints, we use the storage depletion factor $\text{dep}^{\text{storage}}_{c,s}$ and the storage injection factor

$\text{inj}^{\text{storage}}_{c,s}$.

$$\max_{cap^{\text{storage}}_{n,c,s},\, out^{\text{storage}}_{m,n,c,s},\, in^{\text{storage}}_{m,n,c,s}} \Pi^{\text{storage}}_{n,c}(cap^{\text{storage}}_{n,c,s},\, out^{\text{storage}}_{m,n,c,s},\, in^{\text{storage}}_{m,n,c,s}) =$$
$$\sum_{m,s} \left(\left(out^{\text{storage}}_{m,n,c,s} - in^{\text{storage}}_{m,n,c,s}\right) \cdot \beta_{m,n,c}\right)$$
$$- \sum_{s} \left(cap^{\text{storage}}_{n,c,s} \cdot \left(\text{sic}^{\text{storage}}_{c,s} \cdot \text{a}^{\text{storage}}_{n,c,s} + \text{fom}^{\text{storage}}_{n,s}\right)\right)$$
$$- \sum_{m,s} \left(in^{\text{storage}}_{m,n,c,s} \cdot \text{vcost}^{storage}_{n,c,s}\right) \quad \forall n, c. \tag{16}$$

s.t.

$$lvl^{\text{storage}}_{m,n,c,s} + in^{\text{storage}}_{m,n,c,s} - out^{\text{storage}}_{m,n,c,s} - lvl^{\text{storage}}_{m+1,n,c,s} = 0 \quad (\nu_{m,n,c,s}) \quad \forall m, n, c, s. \tag{17}$$

$$cap^{\text{storage}}_{n,c,s} - lvl^{\text{storage}}_{m,n,c,s} \geq 0 \quad (\phi_{m,n,c,s}) \quad \forall m, n, c, s. \tag{18}$$

$$\text{pot}^{\text{storage}}_{n,c,s} - cap^{\text{storage}}_{n,c,s} \geq 0 \quad (\chi_{n,c,s}) \quad \forall n, c, s. \tag{19}$$

$$cap^{\text{storage}}_{n,c,s} \cdot \text{dep}^{\text{storage}}_{c,s} - out^{\text{storage}}_{m,n,c,s} \geq 0 \quad (\psi_{m,n,c,s}) \quad \forall m, n, c, s. \tag{20}$$

$$cap^{\text{storage}}_{n,c,s} \cdot \text{inj}^{\text{storage}}_{c,s} - in^{\text{storage}}_{m,n,c,s} \geq 0 \quad (\xi_{m,n,c,s}) \quad \forall m, n, c, s. \tag{21}$$

J. Market clearing conditions

ERIKSSON consists of the first-order conditions from the previously described optimization problems of the players and two market clearing conditions. A schematic is shown in Figure 1. We introduce Equation 22 as the physical balance of the exporter. The physical balance condition ensures sale by an exporter causes production and physical flows. For every exporter e, at every node n, produced and traded commodities c match physical flows in any month m. The parameter $\text{loss}_{n,n1,c,t}$ considers losses that occur during liquefaction, shipping (boil-off), regasification, and pipeline transportation. The dual variable $\lambda_{m,e,n,c}$ can be interpreted as the supply costs for an exporter e to supply node n with the commodity c in the month m. $\lambda_{m,e,n,c}$ includes production costs as well as transportation costs.

$$\sum_{r,p} (prod_{n,c,r,p} \cdot \text{avail}_{m,n,c,r,p}) + \sum_{n1,t} \left(flow_{m,e,n1,n,c,t} - \frac{flow_{m,e,n,n1,c,t}}{1 - \text{loss}_{n,n1,c,t}}\right) - sell_{m,e,n,c} + buy_{m,e=\text{arbitrageur},n,c} = 0 \quad (\lambda_{m,e,n,c}) \quad \forall m, e, n, c. \tag{22}$$

Equation 23 is the domestic market clearing condition ensuring that supply and demand are matched. At every node n, the demand of every commodity c, as well as the conversion of c to other commodities $c1$ and the storage injection and depletion, have to be served by sale, conversion from other commodities $c1$ to c, and storage depletion in any month m. We calculate price-elastic demand as a linear demand function based on the reference demand $\text{demand}^{\text{ref}}_{m,n,c}$, the reference price $\text{p}^{\text{ref}}_{n,c}$, the market clearing price $\beta_{m,n,c}$, and the $\text{slope}_{m,n,c}$ which results from the price elasticity. The dual variable $\beta_{m,n,c}$ represents the market clearing price for commodity c at node n in month m.

$$\sum_{e,n1} sell_{m,e,n,c} - buy_{m,e=\text{arbitrageur},n,c} + \sum_{s} \left(out^{\text{storage}}_{m,n,c,s} - in^{\text{storage}}_{m,n,c,s}\right) + \sum_{c1} \left(con_{m,n,c1,c} - \frac{con_{m,n,c,c1}}{\text{eff}^{\text{conversion}}_{c,c1}}\right) - \left(\text{demand}^{\text{ref}}_{m,n,c} + \frac{\beta_{m,n,c} - \text{p}^{\text{ref}}_{n,c}}{\text{slope}_{m,n,c}}\right) = 0 \quad (\beta_{m,n,c}) \quad \forall m, n, c. \tag{23}$$

K. Implementation

We derive the model equations by setting up the Karush-Kuhn-Tucker conditions for optimality for the optimization problem of each player. E shows how we derive the first-order conditions, F shows the final model equations. The model is implemented in GAMS and solved with the PATH solver [16].

III. Limitations

ERIKSSON is a multi-commodity partial equilibrium model, focusing on green hydrogen and derivatives markets in isolation due to model complexity constraints. This approach allows for greater detail than a general equilibrium model, though it overlooks interactions with other markets, such as electricity, iron, steel, and chemicals. In ERIKSSON, we represent different equilibria through supply and demand scenarios but cannot capture feedback loops between hydrogen and other markets. Thus, a partial equilibrium does not necessarily represent a general equilibrium.

IV. Outlook

We introduced the mathematical description of ERIKSSON, a global trade model for green hydrogen and hydrogen derivatives, simulating trade and investment decisions in a future global market. ERIKSSON is implemented as a MCP and represents trade as Cournot competition between oligopolistic exporters and other players. We introduce new features depicting the characteristics of hydrogen and hydrogen derivative markets. Among these are the conversion and competition between multiple commodities, temporal availability of RES production, and losses. The current version models one year in a monthly temporal resolution with greenfield investments at the beginning of the year. Thus, we think the current version is best suited for long-term analysis. ERIKSSON could be used for short-term or long-term market ramp-up investigations if expanded to cover multiple years and existing production and infrastructure capacities.

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APPENDIX

A. Literature review

TABLE I
CLASSIFICATION OF EXISTING GREEN ENERGY COMMODITY MARKET MODELS

Study	Spacial scope	Spacial structure	Model nodes	Multiple commodities	Strategic behavior	Demand elasticity	Seasonality	Detailed supply
Antweiler and Schlund [9]	Global	Country	48		✓			
Barner [11]	Global	Regions	26	✓	✓	✓	✓	
Dupke et al. [10]	Global	Regions	6		✓	✓		
Guo et al. [4]	China	Regional	24				✓	
Ikonnikova et al. [6]	Global	Country	140					
Ortmann et al. [7]	Global	Regions	37					✓
Schönfisch [5]	Global	Country	97	✓				✓
Zhang et al. [8]	Global	Regions	29	✓				
This paper	Global	Sub-Country	147[a]	✓	✓	✓	✓	✓

[a]The model includes additional harbor nodes and shipping hubs, which are only transport-relevant. For the sake of comparability, we only count market nodes

B. Schematic of the model

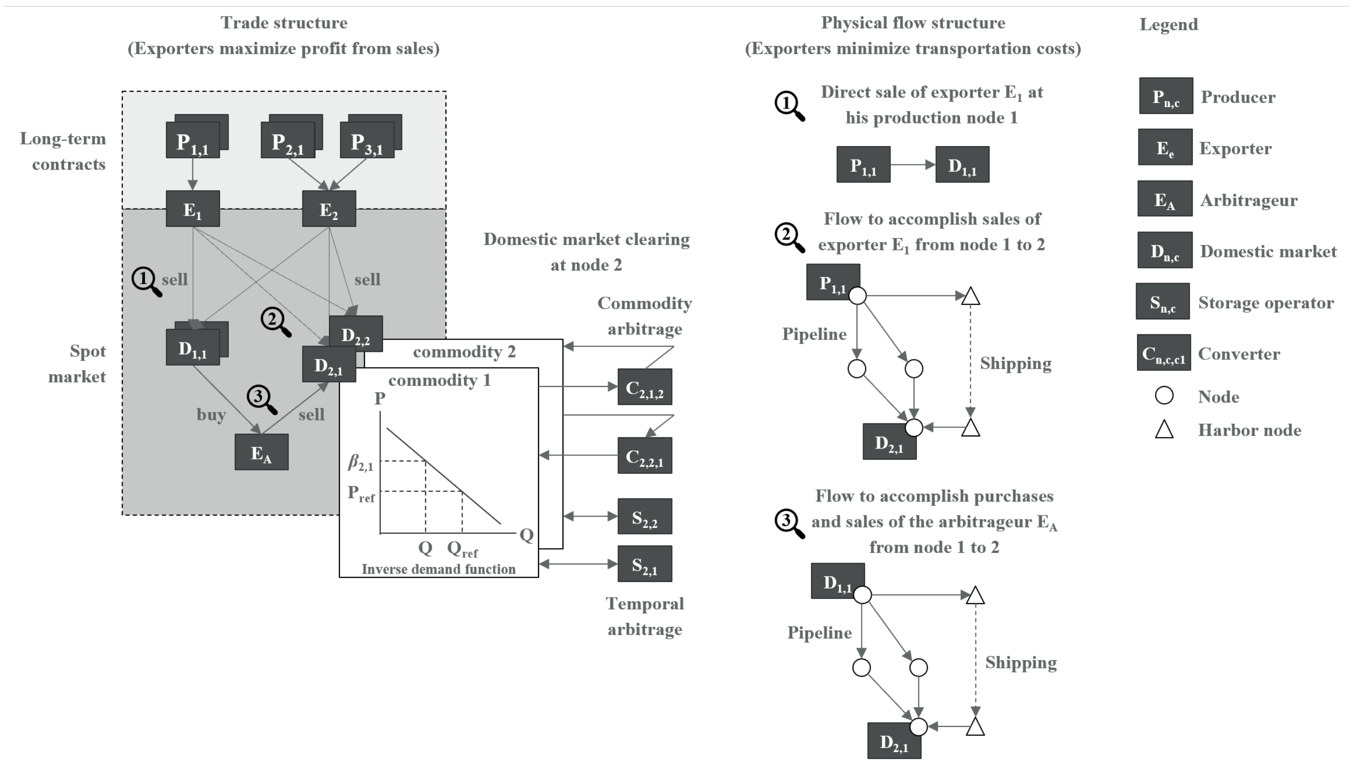

Fig. 1. Schematic of ERIKSSON

C. Spacial structure

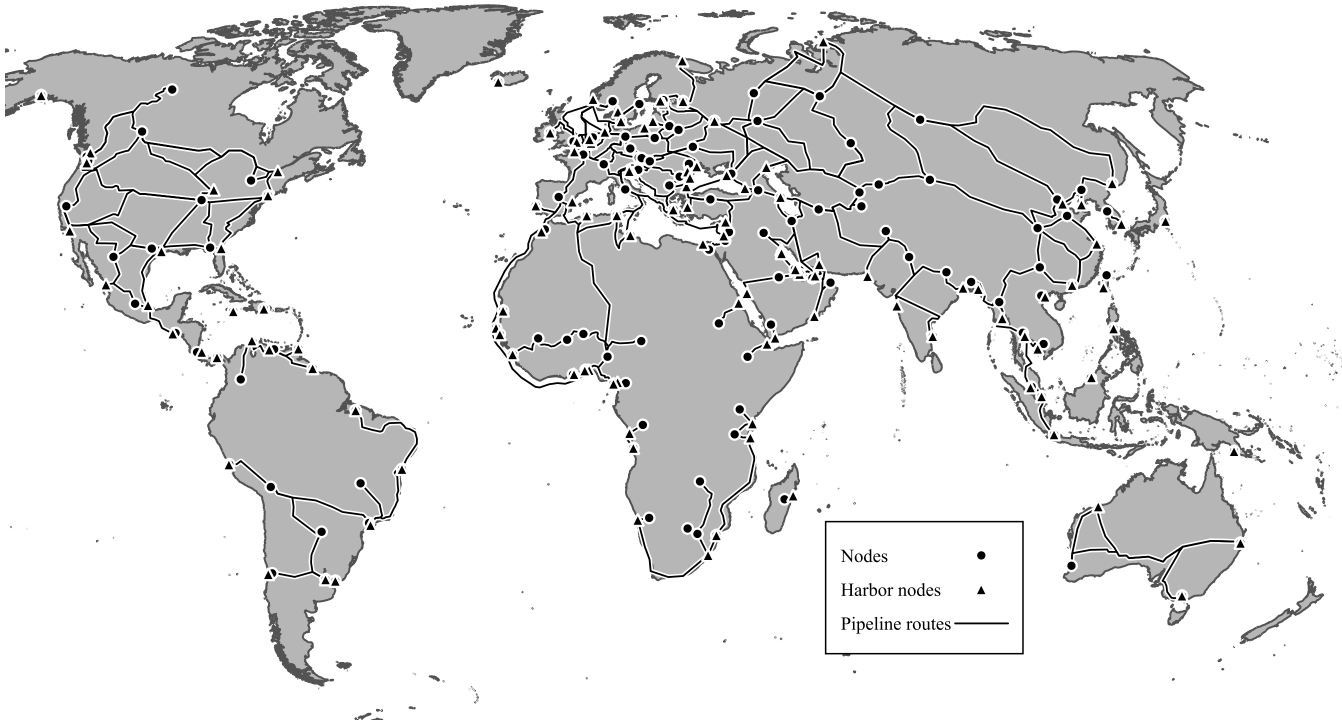

Fig. 2. Map of the model's spacial structure

D. Mathematical notation used in ERIKSSON

TABLE II: Mathematical notation used in ERIKSSON

Sets	
$m \in M$	months per year
$n \in N$	nodes
$h \in H \subset N$	harbor nodes
$v \in V \in (N \times N)$	vertices i.e routes between nodes
$V^{ship} \subset V$	shipping routes
$V^{pipeline} \subset V$	pipeline routes
$e \in E$	exporter
$N^{e} \subset N$	nodes where exporter is active
$c \in C$	commodities
$c \times c \in CT$	Converter types $r \in R$
RES potential classes	
$p \in P$	delivery profiles
$t \in T$	transport route
$s \in S$	storage types
Primal variables	
$prod_{n,c,r,p}$	produced commodity volumes
$sell_{m,e,n,c}$	sold commodity volumes
$flow_{m,e,n,n1,c,t}$	physical commodity flows
$buy_{m,e,n,c}$	bought commodity volumes
$cap^{\text{pipeline}}_{n,n1,c}$	annual investment into pipeline capacity
cap^{ship}_{c}	annual investment into shipping capacity
$cap^{\text{export}}_{h,c}$	annual investment into export terminal capacity
$cap^{\text{import}}_{h,c}$	annual investment into import terminal capacity
$cap^{\text{conversion}}_{n,c,c1}$	annual investment into conversion capacity
$con_{m,n,c,c1}$	converted commodity volumes
$cap^{\text{storage}}_{n,c,s}$	annual investment into storage capacity
$out^{\text{storage}}_{m,n,c,s}$	depleted commodity volume
$in^{\text{storage}}_{m,n,c,s}$	injected commodity volume
$lvl^{\text{storage}}_{m,n,c,s}$	storage level
Dual variables	
$\alpha_{n,r}$	RES potential scarcity rent or marginal benefit of one additional unit of RES potential to the producer
$\gamma_{m,n,n1,c}$	pipeline congestion rent or marginal benefit of one additional unit of pipeline capacity to the exporter
$\delta_{m,c}$	shipping congestion rent or marginal benefit of one additional unit of ship capacity to the exporter
$\eta_{m,h,c}$	export terminal congestion rent or marginal benefit of one additional unit of export terminal capacity to the exporter
$\theta_{m,h,c}$	import terminal congestion rent or marginal benefit of one additional unit of import terminal capacity to the exporter
$\mu_{n,r}$	conversion capacity scarcity rent or marginal benefit of one additional unit of conversion capacity to the converter
$\nu_{m,n,c,s}$	conversion capacity scarcity rent or marginal benefit of one additional unit of conversion capacity to the converter
$\phi_{m,n,c,s}$	storage capacity scarcity rent or marginal benefit of one additional unit of storage capacity to the storage operator
$\chi_{n,c,s}$	storage potential scarcity rent or marginal benefit of one additional unit of storage potential to the storage operator
$\psi_{m,n,c,s}$	storage depletion scarcity rent or marginal benefit of one additional unit of storage depletion capacity to the storage operator

$\xi_{m,n,c,s}$	injection depletion scarcity rent or marginal benefit of one additional unit of storage injection capacity to the storage operator
$\beta_{m,n,c}$	price to acquire one additional unit of commodity
$\lambda_{m,e,n,c}$	supply costs for an exporter to supply one additional unit of commodity
Parameters	
$\text{cost}^{\text{producer}}$	production costs
$\text{pot}_{n,r}^{production}$	RES potential
$\text{eff}_{n,c,r,p}^{production}$	production efficiency, including curtailment
$\text{avail}_{m,n,c,r,p}$	availability of production
$\text{demand}_{m,n,c}^{\text{ref}}$	reference demand
$\text{p}_{n,c}^{\text{ref}}$	reference price
$\text{slope}_{m,n,c}$	slope of the demand function
$\text{cv}_{e,n}$	conjectural variation
$\text{vcost}_{n,n1,c,t}^{\text{transport}}$	variable costs of the transport method between two nodes
$\text{sic}_{c}^{\text{pipeline}}$	specific investment costs for pipelines
$\text{a}_{c}^{\text{pipeline}}$	annuity for pipelines
$\text{fom}_{c}^{\text{pipeline}}$	FOM costs for pipelines
$\text{dist}_{n,n1}^{pipeline}$	distance for pipelines
$\text{sic}_{c}^{\text{ship}}$	specific investment costs for ships
$\text{a}_{c}^{\text{ship}}$	annuity for ships
$\text{fom}_{c}^{\text{ship}}$	FOM costs for ships
speed	speed of ships
$\text{dist}_{h,h1}^{\text{ship}}$	distance for shipping
$\text{sic}_{c}^{\text{export}}$	specific investment costs for export terminals
$\text{a}_{c}^{\text{export}}$	annuity for export terminals
$\text{fom}_{c}^{\text{export}}$	FOM costs for export terminals
$\text{vcost}_{h,c,t}^{\text{export}}$	variable costs of the export terminal
$\text{sic}_{c}^{\text{import}}$	specific investment costs for import terminals
$\text{a}_{c}^{\text{import}}$	annuity for import terminals
$\text{fom}_{c}^{\text{import}}$	FOM costs for import terminals
$\text{vcost}_{h,c,t}^{\text{import}}$	variable costs of the import terminal
$\text{sic}_{n,c,c1}^{\text{converter}}$	specific investment costs for conversion
$\text{a}_{n,c,c1}^{\text{converter}}$	annuity for conversion
$\text{fom}_{n,c,c1}^{\text{converter}}$	FOM costs for conversion
$\text{vcost}_{n,c,c1}^{\text{conversion}}$	variable costs of the conversion between two commodities
$\text{eff}_{c,c1}^{\text{converter}}$	conversion efficiency
$\text{sic}_{c,s}^{\text{storage}}$	specific investment costs for storage
$\text{a}_{n,c,s}^{\text{storage}}$	annuity for storage
$\text{fom}_{n,s}^{\text{storage}}$	FOM costs for storage
$\text{vcost}_{n,c,s}^{\text{storage}}$	variable costs for one storage cycle
$\text{pot}_{n,c,s}^{\text{storage}}$	storage potential
$\text{dep}_{c,s}^{\text{storage}}$	depletion capacity per storage capacity
$\text{inj}_{c,s}^{\text{storage}}$	injection capacity per storage capacity
$\text{loss}_{n,n1,c,t}$	losses for transport, including liquefaction and regasification

E. Derivation of the first order conditions

1) The producer's problem: The Lagrangian of the producer's problem is defined as:

$$\mathcal{L}^{\text{producer}}_{e,n,c}(prod_{n,c,r,p}) = \sum_{m,r,p} prod_{n,c,r,p} \cdot \text{avail}_{m,n,c,r,p} \cdot \left(\text{cost}^{\text{production}}_{n,c,r,p} - \lambda_{m,e,n,c}\right) - \sum_{r} \alpha_{n,r} \cdot \left(\text{pot}^{production}_{n,r} - \sum_{m,p} \frac{prod_{n,c,r,p} \cdot \text{avail}_{m,n,c,r,p}}{\text{eff}^{production}_{n,c,r,p}}\right) \quad \forall n \in N^e, c. \tag{24}$$

The first-order optimality condition of the producer's profit maximization problem is defined by the first partial derivative of the Lagrangian with respect to the positive variable $prod_{n,c,r,p}$:

$$\frac{\partial \mathcal{L}^{\text{producer}}_{e,n,c}}{\partial prod_{n,c,r,p}} = \text{cost}^{\text{production}}_{n,c,r,p} - \sum_{m,e} (\lambda_{m,e,n,c} \cdot \text{avail}_{m,n,c,r,p}) + \frac{\alpha_{n,r}}{\text{eff}^{production}_{n,c,r,p}} \geq 0 \quad \forall n, c, r, p. \tag{25}$$

2) The exporter's problems: The exporter has two problems, the profit maximization problem and transport cost minimization problem. The Lagrangian of the exporter's profit maximization problem is defined as:

$$\mathcal{L}^{\text{exporter-profit}}_{e}(sell_{m,e,n,c}) = \sum_{m,n,c} \Big[sell_{m,e,n,c} \cdot \Big(\lambda_{m,e,n,c} - (1-\text{cv}_{e,n}) \cdot \beta_{m,n,c} - \text{cv}_{e,n} \cdot \Big(\text{p}^{\text{ref}}_{n,c} + \text{slope}_{m,n,c} \cdot (\sum_{e} sell_{m,e,n,c} - buy_{m,e=\text{arbitrageur},n,c})\Big)\Big)\Big] \quad \forall e. \tag{26}$$

The Lagrangian is differentiated with respect to the positive variable $sell_{m,e,n,c}$, yielding the following equation:

$$\frac{\partial \mathcal{L}^{\text{exporter-profit}}_{e}}{\partial sell_{m,e,n,c}} = \lambda_{m,e,n,c} - (1-\text{cv}_{e,n}) \cdot \beta_{m,n,c} - \text{cv}_{e,n} \cdot \Big(\text{p}^{\text{ref}}_{n,c} + \text{slope}_{m,n,c} \cdot \Big(\sum_{e} sell_{m,e,n,c} + sell_{m,e,n,c} - buy_{m,e=\text{arbitrageur},n,c}\Big)\Big) \geq 0 \quad \forall m, e, n, c. \tag{27}$$

The inverse demand function is defined as:

$$\beta_{m,n,c} = \text{p}^{\text{ref}}_{n,c} + \text{slope}_{m,n,c} \cdot \Big(\sum_{e} sell_{m,e,n,c} - buy_{m,e=\text{arbitrageur},n,c}\Big) \quad \forall m, n, c. \tag{28}$$

Transforming Equation 27 several times and inserting the inverse demand function Equation 28 yields the first-order optimality condition of the exporter's profit maximization problem:

$$\begin{aligned}
&\frac{\partial \mathcal{L}^{\text{exporter-profit}}}{\partial sell_{m,e,n,c}} = \\
&\lambda_{m,e,n,c} - (1-\text{cv}_{e,n}) \cdot \beta_{m,n,c} - \text{cv}_{e,n} \cdot \Big(\text{p}^{\text{ref}}_{n,c} + \text{slope}_{m,n,c} \cdot \Big(\sum_{e} sell_{m,e,n,c} + sell_{m,e,n,c} - buy_{m,e=\text{arbitrageur},n,c}\Big)\Big) = \\
&\lambda_{m,e,n,c} - (1-\text{cv}_{e,n}) \cdot \beta_{m,n,c} - \text{cv}_{e,n} \cdot \Big(\text{p}^{\text{ref}}_{n,c} + \text{slope}_{m,n,c} \cdot (\sum_{e} sell_{m,e,n,c} - buy_{m,e=\text{arbitrageur},n,c})\Big) - \text{cv}_{e,n} \cdot \text{slope}_{m,n,c} \cdot sell_{m,e,n,c} = \\
&\lambda_{m,e,n,c} - (1-\text{cv}_{e,n}) \cdot \beta_{m,n,c} - \text{cv}_{e,n} \cdot \beta_{m,n,c} - \text{cv}_{e,n} \cdot \text{slope}_{m,n,c} \cdot sell_{m,e,n,c} = \\
&\lambda_{m,e,n,c} - \beta_{m,n,c} + \text{cv}_{e,n} \cdot \beta_{m,n,c} - \text{cv}_{e,n} \cdot \beta_{m,n,c} - \text{cv}_{e,n} \cdot \text{slope}_{m,n,c} \cdot sell_{m,e,n,c} = \\
&\lambda_{m,e,n,c} - \beta_{m,n,c} - \text{cv}_{e,n} \cdot \text{slope}_{m,n,c} \cdot sell_{m,e,n,c} \geq 0 \quad \forall m, e, n, c.
\end{aligned} \tag{29}$$

The Lagrangian of the exporter's transport cost minimization problem is defined as:

$$\begin{aligned}
&\mathcal{L}^{\text{exporter-transport}}_{e}(flow_{m,e,n,n1,c,t}) = \\
&\sum_{m,(n \times n1) \in V,c,t} \Big(flow_{m,e,n,n1,c,t} \cdot \Big[\Big(\text{vcost}^{\text{transport}}_{n,n1,c,t} + \text{vcost}^{\text{export}}_{n,c,t} + \text{vcost}^{\text{import}}_{n1,c,t}\Big) - \big(\lambda_{m,e,n1,c} - \lambda_{m,e,n,c}\big)\Big]\Big) \\
&- \sum_{m,(n \times n1) \in V^{pipeline},c} \Big(\gamma_{m,n,n1,c} \cdot \Big(cap^{\text{pipeline}}_{n,n1,c} - \sum_{e,t=\text{pipeline}} flow_{m,e,n,n1,c,t}\Big)\Big) \\
&- \sum_{m,c} \Big(\delta_{m,c} \cdot cap^{\text{ship}}_{c} \cdot \frac{8760}{12} \cdot \text{speed} - \sum_{e,(h \times h1) \in V^{ship},t=\text{ship}} \Big[2 \cdot flow_{m,e,h,h1,c,t} \cdot \text{dist}^{\text{ship}}_{h,h1}\Big]\Big) \\
&- \sum_{m,c} \Big(\eta_{m,h,c} \cdot \Big(cap^{\text{exp}}_{h,c} - \sum_{e,h1,t=\text{ship}} flow_{m,e,h,h1,c,t}\Big)\Big) \\
&- \sum_{m,c} \Big(\theta_{m,h1,c} \cdot \Big(cap^{\text{imp}}_{1h,c} - \sum_{e,h,t=\text{ship}} flow_{m,e,h,h1,c,t}\Big)\Big) \quad \forall e.
\end{aligned} \tag{30}$$

The first-order optimality condition of the exporter's transport cost minimization problem is defined by the first partial derivative of the Lagrangian with respect to the positive variable $flow_{m,e,n,n1,c,t}$:

$$\frac{\partial \mathcal{L}_e^{\text{exporter-transport}}}{\partial flow_{m,e,n,n1,c,t}} = \text{vcost}_{n,n1,c,t}^{\text{transport}} + \text{vcost}_{n,c,t}^{\text{export}} + \text{vcost}_{n1,c,t}^{\text{import}} - \left(\lambda_{m,e,n1,c} - \lambda_{m,e,n,c}\right) + \gamma_{m,n,n1,c} + \delta_{m,c} \cdot 2 \cdot \text{dist}_{h,h1}^{\text{ship}} + \eta_{m,h,c} + \theta_{m,h1,c} \geq 0 \quad \forall m, e, (n \times n1) \in V, c, t. \tag{31}$$

3) The buyer's problem: The Lagrangian of the buyer's problem is defined as:

$$\mathcal{L}_e^{\text{buyer}}(buy_{m,e,n,c}) = \sum_{m,n,c} \Big[buy_{m,e,n,c} \cdot (\beta_{m,n,c} - \lambda_{m,e,n,c})\Big], \quad e = \text{arbitrageur}. \tag{32}$$

The first-order optimality conditions of the buyer's profit maximization function are defined by the first partial derivative of the Lagrangian with respect to the positive variable $buy_{m,e,n,c}$:

$$\frac{\partial \mathcal{L}_e^{\text{buyer}}}{\partial buy_{m,e,n,c}} = \beta_{m,n,c} - \lambda_{m,e,n,c} \geq 0, \quad \forall m, n, c, e = \text{arbitrageur}. \tag{33}$$

4) The pipeline operator's problem: The Lagrangian of the pipeline operator's problem is defined as:

$$\mathcal{L}_{n,n1,c}^{\text{pipeline}}(cap_{n,n1,c}^{\text{pipeline}}) = cap_{n,n1,c}^{\text{pipeline}} \cdot \Big(\text{sic}_c^{\text{pipeline}} \cdot \text{dist}_{n,n1}^{pipeline} \cdot (\text{a}_c^{\text{pipeline}} + \text{fom}_c^{\text{pipeline}}) - \sum_m \gamma_{m,n,n1,c}\Big) \quad \forall (n \times n1) \in V^{pipeline}, c. \tag{34}$$

The first-order optimality condition of the pipeline operator's profit maximization problem is defined by the first partial derivative of the Lagrangian with respect to the positive variable $cap_{n,n1,c}^{\text{pipeline}}$:

$$\frac{\partial \mathcal{L}_{n,n1,c}^{\text{pipeline}}}{\partial cap_{n,n1,c}^{\text{pipeline}}} = \text{sic}_c^{\text{pipeline}} \cdot \text{dist}_{n,n1}^{pipeline} \cdot (\text{a}_c^{\text{pipeline}} + \text{fom}_c^{\text{pipeline}}) - \sum_m \gamma_{m,n,n1,c} \geq 0 \quad \forall (n \times n1) \in V^{pipeline}, c. \tag{35}$$

5) The ship operator's problem: The Lagrangian of the ship operator's problem is defined as:

$$\mathcal{L}_c^{\text{ship}}(cap_c^{\text{ship}}) = cap_c^{\text{ship}} \cdot \Big(\text{sic}_c^{\text{ship}} \cdot (\text{a}_c^{\text{ship}} + \text{fom}_c^{\text{ship}}) - \sum_m \delta_{m,c}\Big) \quad \forall c. \tag{36}$$

The first-order optimality condition of the ship operator's profit maximization problem is defined by the first partial derivative of the Lagrangian with respect to the positive variable cap_c^{ship}:

$$\frac{\partial \mathcal{L}_c^{\text{ship}}}{\partial cap_c^{\text{ship}}} = \text{sic}_c^{\text{ship}} \cdot (\text{a}_c^{\text{ship}} + \text{fom}_c^{\text{ship}}) - \sum_m \delta_{m,c} \geq 0 \quad \forall c. \tag{37}$$

6) The export terminal operator's problem: The Lagrangian of the export terminal operator's problem is defined as:

$$\mathcal{L}_{h,c}^{\text{export}}(cap_{h,c}^{\text{export}}) = cap_{h,c}^{\text{export}} \cdot \Big(\text{sic}_c^{\text{export}} \cdot (\text{a}_c^{\text{export}} + \text{fom}_c^{\text{export}}) - \sum_m \eta_{m,h,c}\Big) \quad \forall h, c. \tag{38}$$

The first-order optimality condition of the export terminal operator's profit maximization problem is defined by the first partial derivative of the Lagrangian with respect to the positive variable $cap_{h,c}^{\text{export}}$:

$$\frac{\partial \mathcal{L}_{h,c}^{\text{export}}}{\partial cap_{h,c}^{\text{export}}} = \text{sic}_c^{\text{export}} \cdot (\text{a}_c^{\text{export}} + \text{fom}_c^{\text{export}}) - \sum_m \eta_{m,h,c} \geq 0 \quad \forall h, c. \tag{39}$$

7) The import terminal operator's problem: The Lagrangian of the import terminal operator's problem is defined as:

$$\mathcal{L}_{h,c}^{\text{import}}(cap_{h,c}^{\text{import}}) = cap_{h,c}^{\text{import}} \cdot \Big(\text{sic}_c^{\text{import}} \cdot (\text{a}_c^{\text{import}} + \text{fom}_c^{\text{import}}) - \sum_m \theta_{m,h,c}\Big) \quad \forall h, c. \tag{40}$$

The first-order optimality condition of the import terminal operator's profit maximization problem is defined by the first partial derivative of the Lagrangian with respect to the positive variable $cap_{h,c}^{\text{import}}$:

$$\frac{\partial \mathcal{L}_{h,c}^{\text{import}}}{\partial cap_{h,c}^{\text{import}}} = \text{sic}_c^{\text{import}} \cdot (\text{a}_c^{\text{import}} + \text{fom}_c^{\text{import}}) - \sum_m \theta_{m,h,c} \geq 0 \quad \forall h, c. \tag{41}$$

8) The converter's problem: The Lagrangian of the converter's problem is defined as:

$$\mathcal{L}_{n,c,c1}^{\text{conversion}}(cap_{n,c,c1}^{\text{conversion}}, con_{m,n,c,c1}) = cap_{n,c,c1}^{\text{conversion}} \cdot \Big(\text{sic}_{n,c,c1}^{\text{conversion}} \cdot \text{a}_{n,c,c1}^{\text{conversion}} + \text{fom}_{n,c,c1}^{\text{conversion}}\Big) + \sum_m \left[con_{m,n,c,c1} \cdot \text{vcost}_{n,c,c1}^{\text{conversion}}\right] - \sum_m \left[con_{m,n,c,c1} \cdot \left(\beta_{m,n,c1} - \frac{\beta_{m,n,c}}{\text{eff}_{c,c1}^{\text{conversion}}}\right)\right] - \sum_m \left[\mu_{m,n,c,c1} \cdot \left(cap_{n,c,c1}^{\text{conversion}} - con_{m,n,c,c1}\right)\right] \quad \forall n, (c \times c1) \in CT. \tag{42}$$

The first-order optimality conditions of the converters's profit maximization problem are defined by the first partial derivative of the Lagrangian with respect to the positive variables $cap_{n,c,c1}^{\text{conversion}}$ and $con_{m,n,c,c1}$:

$$\frac{\partial \mathcal{L}^{\text{conversion}}_{n,c,c1}}{\partial cap^{\text{conversion}}_{n,c,c1}} = \text{sic}^{\text{conversion}}_{n,c,c1} \cdot \text{a}^{\text{conversion}}_{n,c,c1} + \text{fom}^{\text{conversion}}_{n,c,c1} - \sum_{m} \mu_{m,n,c,c1} \geq 0 \quad \forall n, (c \times c1) \in CT. \tag{43}$$

$$\frac{\partial \mathcal{L}^{\text{conversion}}_{n,c,c1}}{\partial con_{m,n,c,c1}} = \frac{\beta_{m,n,c}}{\text{eff}^{\text{conversion}}_{c,c1}} - \beta_{m,n,c1} + \text{vcost}^{\text{conversion}}_{n,c,c1} + \mu_{m,n,c,c1} \geq 0 \quad \forall m, n, (c \times c1) \in CT. \tag{44}$$

9) The storage operator's problem: The Lagrangian of the storage operator's problem is defined as:

$$\begin{aligned}
&\mathcal{L}^{\text{storage}}_{n,c}(cap^{\text{storage}}_{n,c,s},\ out^{\text{storage}}_{m,n,c,s},\ in^{\text{storage}}_{m,n,c,s}) = \\
&+ \sum_{s} \Big[cap^{\text{storage}}_{n,c,s} \cdot \big(\text{sic}^{\text{storage}}_{c,s} \cdot \text{a}^{\text{storage}}_{n,c,s} + \text{fom}^{\text{storage}}_{n,s} \big) \Big] \\
&+ \sum_{m,s} \Big[in^{\text{storage}}_{m,n,c,s} \cdot \text{vcost}^{storage}_{n,c,s} \Big] \\
&- \sum_{m,s} \Big[\big(out^{\text{storage}}_{m,n,c,s} - in^{\text{storage}}_{m,n,c,s} \big) \cdot \beta_{m,n,c} \Big] \\
&- \sum_{m,s} \Big[\nu_{m,n,c,s} \cdot \big(lvl^{\text{storage}}_{m,n,c,s} + in^{\text{storage}}_{m,n,c,s} - out^{\text{storage}}_{m,n,c,s} - lvl^{\text{storage}}_{m+1,n,c,s} \big) \Big] \\
&- \sum_{m,s} \Big[\phi_{m,n,c,s} \cdot \big(cap^{\text{storage}}_{n,c,s} - lvl^{\text{storage}}_{m,n,c,s} \big) \Big] \\
&- \sum_{s} \Big[\chi_{n,c,s} \cdot \big(\text{pot}^{\text{storage}}_{n,c,s} - cap^{\text{storage}}_{n,c,s} \big) \Big] \\
&- \sum_{m,s} \Big[\psi_{m,n,c,s} \cdot \big(cap^{\text{storage}}_{n,c,s} \cdot \text{dep}^{\text{storage}}_{c,s} - out^{\text{storage}}_{m,n,c,s} \big) \Big] \\
&- \sum_{m,s} \Big[\xi_{m,n,c,s} \cdot \big(cap^{\text{storage}}_{n,c,s} \cdot \text{inj}^{\text{storage}}_{c,s} - in^{\text{storage}}_{m,n,c,s} \big) \Big] \\
&\forall n, c.
\end{aligned} \tag{45}$$

The first-order optimality conditions of the storage operator's profit maximization problem are defined by the first partial derivative of the Lagrangian with respect to the positive variables $cap^{\text{storage}}_{n,c,s}$, $in^{\text{storage}}_{m,n,c,s}$, $out^{\text{storage}}_{m,n,c,s}$ and $lvl^{\text{storage}}_{m,n,c,s}$:

$$\begin{aligned}
&\frac{\partial \mathcal{L}^{\text{storage}}_{n,c}}{\partial cap^{\text{storage}}_{n,c,s}} = \\
&\text{sic}^{\text{storage}}_{c,s} \cdot \text{a}^{\text{storage}}_{n,c,s} + \text{fom}^{\text{storage}}_{n,s} - \sum_{m} \phi_{m,n,c,s} + \chi_{n,c,s} \\
&- \text{inj}^{\text{storage}}_{c,s} \cdot \sum_{m} \xi_{m,n,c,s} - \text{dep}^{\text{storage}}_{c,s} \cdot \sum_{m} \psi_{m,n,c,s} \geq 0 \\
&\forall n, c, s.
\end{aligned} \tag{46}$$

$$\frac{\partial \mathcal{L}^{\text{storage}}_{n,c}}{\partial in^{\text{storage}}_{m,n,c,s}} = \beta_{m,n,c} + \text{vcost}^{storage}_{n,c,s} - \nu_{m,n,c,s} + \xi_{m,n,c,s} \geq 0 \quad \forall m, n, c, s. \tag{47}$$

$$\frac{\partial \mathcal{L}^{\text{storage}}_{n,c}}{\partial out^{\text{storage}}_{m,n,c,s}} = -\beta_{m,n,c} + \nu_{m,n,c,s} + \psi_{m,n,c,s} \geq 0 \quad \forall m, n, c, s. \tag{48}$$

$$\frac{\partial \mathcal{L}^{\text{storage}}_{n,c}}{\partial lvl^{\text{storage}}_{m,n,c,s}} = \phi_{m,n,c,s} - \nu_{m,n,c,s} + \nu_{m+1,n,c,s} \geq 0 \quad \forall m, n, c, s. \tag{49}$$

F. Model equations

This section comprises the Karush-Kuhn-Tucker (KKT) conditions of the model.

1) The producer's problem:

$$\begin{pmatrix} \text{cost}^{\text{production}}_{n,c,r,p} - \sum_{m,e} \lambda_{m,e,n,c} \\ + \frac{\alpha_{n,r}}{\text{eff}^{production}_{n,c,r,p}} \end{pmatrix} \geq 0 \perp prod_{n,c,r,p} \geq 0 \quad \forall n,c,r,p. \tag{50}$$

$$\begin{pmatrix} \text{pot}^{production}_{n,r} - \\ \sum_{m,c,p} \frac{prod_{n,c,r,p} \cdot \text{avail}_{m,n,c,r,p}}{\text{eff}^{production}_{n,c,r,p}} \end{pmatrix} \geq 0 \perp \alpha_{n,r} \geq 0 \quad \forall n,r. \tag{51}$$

2) The exporter's problems:

$$\begin{pmatrix} \lambda_{m,e,n,c} - \beta_{m,n,c} \\ -\text{cv}_{e,n} \cdot \text{slope}_{m,n,c} \cdot sell_{m,e,n,c} \end{pmatrix} \geq 0 \perp sell_{m,e,n,c} \geq 0 \quad \forall m,e,n,c. \tag{52}$$

$$\begin{pmatrix} \text{vcost}^{\text{transport}}_{n,n1,c,t} + \text{vcost}^{\text{export}}_{n,c,t} + \text{vcost}^{\text{import}}_{n1,c,t} \\ -(\lambda_{m,e,n1,c} - \lambda_{m,e,n,c}) \\ +\gamma_{m,n,n1,c} + \delta_{m,c} \cdot 2 \cdot \text{dist}^{\text{ship}}_{h,h1} \\ +\eta_{m,h,c} + \theta_{m,h1,c} \end{pmatrix} \geq 0 \perp flow_{m,e,n,n1,c,t} \geq 0 \quad \forall m,e,(n \times n1) \in V, c, t. \tag{53}$$

$$\left(cap^{\text{pipeline}}_{n,n1,c} - \sum_{e,t=pipeline} flow_{m,e,n,n1,c,t} \right) \geq 0 \perp \gamma_{m,n,n1,c} \geq 0 \quad \forall m,(n \times n1) \in V, c. \tag{54}$$

$$\begin{pmatrix} cap^{\text{ship}}_c \cdot \frac{8760}{12} \cdot \text{speed} \\ - \sum_{e,(h \times h1) \in V^{shipping}, t=ship} \left(2 \cdot flow_{m,e,h,h1,c,t} \cdot \text{dist}^{\text{ship}}_{h,h1} \right) \end{pmatrix} \geq 0 \perp \delta_{m,c} \geq 0 \quad \forall m,c. \tag{55}$$

$$\left(cap^{\text{exp}}_{h,c} - \sum_{e,h1,t=ship} flow_{m,e,h,h1,c,t} \right) \geq 0 \perp \eta_{m,h,c} \geq 0 \quad \forall m,h,c. \tag{56}$$

$$\left(cap^{\text{imp}}_{h,c} - \sum_{e,h1,t} flow_{m,e,h1,h,c,t=ship} \right) \geq 0 \perp \theta_{m,h,c} \geq 0 \quad \forall m,h,c. \tag{57}$$

3) The buyer's problem:

$$\left(\beta_{m,n,c} - \lambda_{m,e,n,c} \right) \geq 0 \perp buy_{m,e,n,c} \geq 0 \quad \forall m,n,c,e = \text{arbitrageur}. \tag{58}$$

4) The pipeline operator's problem:

$$\begin{pmatrix} \text{sic}^{\text{pipeline}}_c \cdot \text{dist}^{pipeline}_{n,n1} \cdot (\text{a}^{\text{pipeline}}_c + \text{fom}^{\text{pipeline}}_c) \\ - \sum_m \gamma_{m,n,n1,c} \end{pmatrix} \geq 0 \perp cap^{\text{pipeline}_{n,n1,c}} \geq 0 \quad \forall (n \times n1) \in V^{pipeline}, c. \tag{59}$$

5) The ship operator's problem:

$$\begin{pmatrix} \text{sic}^{\text{ship}}_c \cdot (\text{a}^{\text{ship}}_c + \text{fom}^{\text{ship}}_c) \\ - \sum_m \delta_{m,c} \end{pmatrix} \geq 0 \perp cap^{\text{ship}_c} \geq 0 \quad \forall c. \tag{60}$$

6) The export terminal operator's problem:

$$\begin{pmatrix} \text{sic}^{\text{export}}_c \cdot (\text{a}^{\text{export}}_c + \text{fom}^{\text{export}}_c) \\ - \sum_m \eta_{m,h,c} \end{pmatrix} \geq 0 \perp cap^{\text{export}}_{h,c} \geq 0 \quad \forall h,c. \tag{61}$$

7) The import terminal operator's problem:

$$\begin{pmatrix} \text{sic}^{\text{import}}_c \cdot (\text{a}^{\text{import}}_c + \text{fom}^{\text{import}}_c) \\ - \sum_m \theta_{m,h,c} \end{pmatrix} \geq 0 \perp cap^{\text{import}}_{h,c} \geq 0 \quad \forall h,c. \tag{62}$$

8) The converter's problem:

$$\begin{pmatrix} \text{sic}^{\text{conversion}}_{n,c,c1} \cdot \text{a}^{\text{conversion}}_{n,c,c1} + \text{fom}^{\text{conversion}}_{n,c,c1} \\ - \sum_m \mu_{m,n,c,c1} \end{pmatrix} \geq 0 \perp cap^{\text{conversion}}_{n,c,c1} \geq 0 \quad \forall n,(c \times c1) \in CT. \tag{63}$$

$$\begin{pmatrix} \frac{\beta_{m,n,c}}{\text{eff}^{\text{conversion}}_{c,c1}} - \beta_{m,n,c1} \\ +\text{vcost}^{\text{conversion}}_{n,c,c1} + \mu_{m,n,c,c1} \end{pmatrix} \geq 0 \perp con_{m,n,c,c1} \geq 0 \quad \forall m,n,(c \times c1) \in CT. \tag{64}$$

$$\left(cap^{\text{conversion}}_{n,c,c1} - con_{m,n,c,c1} \right) \geq 0 \perp \mu_{m,n,c,c1} \geq 0 \quad \forall m,n,(c \times c1) \in CT. \tag{65}$$

9) The storage operator's problem:

$$\begin{pmatrix} \text{sic}^{\text{storage}}_{c,s} \cdot \text{a}^{\text{storage}}_{n,c,s} + \text{fom}^{\text{storage}}_{n,s} \\ - \sum_m \phi_{m,n,c,s} + \chi_{n,c,s} \\ -\text{inj}^{\text{storage}}_{c,s} \cdot \sum_m \xi_{m,n,c,s} \\ -\text{dep}^{\text{storage}}_{c,s} \cdot \sum_m \psi_{m,n,c,s} \end{pmatrix} \geq 0 \perp cap^{\text{storage}}_{n,c,s} \geq 0 \quad \forall n,c,s. \tag{66}$$

$$\begin{pmatrix} \beta_{m,n,c} + \text{vcost}^{storage}_{n,c,s} \\ -\nu_{m,n,c,s} + \xi_{m,n,c,s} \end{pmatrix} \geq 0 \perp in^{\text{storage}}_{m,n,c,s} \geq 0 \quad \forall m,n,c,s. \tag{67}$$

$$\begin{pmatrix} -\beta_{m,n,c} + \nu_{m,n,c,s} \\ +\psi_{m,n,c,s} \end{pmatrix} \geq 0 \perp out^{\text{storage}}_{m,n,c,s} \geq 0 \quad \forall m,n,c,s. \tag{68}$$

$$\begin{pmatrix} \phi_{m,n,c,s} - \nu_{m,n,c,s} \\ +\nu_{m+1,n,c,s} \end{pmatrix} \geq 0 \perp lvl^{\text{storage}}_{m,n,c,s} \geq 0 \quad \forall m,n,c,s.$$

$$(69)$$

$$\begin{pmatrix} lvl^{\text{storage}}_{m,n,c,s} + in^{\text{storage}}_{m,n,c,s} \\ -out^{\text{storage}}_{m,n,c,s} - lvl^{\text{storage}}_{m+1,n,c,s} \end{pmatrix} \quad \begin{matrix} = 0 \perp \nu_{m,n,c,s} \\ \forall m,n,c,s. \end{matrix} \tag{70}$$

$$\begin{pmatrix} cap^{\text{storage}}_{n,c,s} - lvl^{\text{storage}}_{m,n,c,s} \end{pmatrix} \quad \begin{matrix} \geq 0 \perp \phi_{m,n,c,s} \geq 0 \\ \forall m,n,c,s. \end{matrix} \tag{71}$$

$$\begin{pmatrix} \text{pot}^{\text{storage}}_{n,c,s} - cap^{\text{storage}}_{n,c,s} \end{pmatrix} \quad \begin{matrix} \geq 0 \perp \chi_{n,c,s} \geq 0 \\ \forall n,c,s. \end{matrix} \tag{72}$$

$$\begin{pmatrix} cap^{\text{storage}}_{n,c,s} \cdot \text{dep}^{\text{storage}}_{c,s} \\ -out^{\text{storage}}_{m,n,c,s} \end{pmatrix} \quad \begin{matrix} \geq 0 \perp \psi_{m,n,c,s} \geq 0 \\ \forall m,n,c,s. \end{matrix} \tag{73}$$

$$\begin{pmatrix} cap^{\text{storage}}_{n,c,s} \cdot \text{inj}^{\text{storage}}_{c,s} \\ -in^{\text{storage}}_{m,n,c,s} \end{pmatrix} \quad \begin{matrix} \geq 0 \perp \xi_{m,n,c,s} \geq 0 \\ \forall m,n,c,s. \end{matrix} \tag{74}$$

10) Market clearing conditions:

$$\begin{pmatrix} \sum_{r,p} \left(prod_{n,c,r,p} \cdot \text{avail}_{m,n,c,r,p} \right) \\ + \sum_{n1,t} \left(flow_{m,e,n1,n,c,t} - \frac{flow_{m,e,n,n1,c,t}}{1-\text{loss}_{n,n1,c,t}} \right) \\ -sell_{m,e,n,c} + buy_{m,e=\text{arbitrageur},n,c} \end{pmatrix} \quad \begin{matrix} = 0 \perp \lambda_{m,e,n,c} \\ \forall m,e,n,c. \end{matrix} \tag{75}$$

$$\begin{pmatrix} \sum_{e,n1} sell_{m,e,n,c} - buy_{m,e=\text{arbitrageur},n,c} \\ + \sum_{s} \left(out^{\text{storage}}_{m,n,c,s} - in^{\text{storage}}_{m,n,c,s} \right) \\ + \sum_{c1} \left(con_{m,n,c1,c} - \frac{con_{m,n,c,c1}}{\text{eff}^{\text{conversion}}_{c,c1}} \right) \\ - \left(\text{demand}^{\text{ref}}_{m,n,c} + \frac{\beta_{m,n,c} - \text{p}^{\text{ref}}_{n,c}}{\text{slope}_{m,n,c}} \right) \end{pmatrix} \quad \begin{matrix} = 0 \perp \beta_{m,n,c} \\ \forall m,n,c. \end{matrix} \tag{76}$$