Integration of Block, Linked and Loop Bids in a Market Coupling Algorithm

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***Abstract*—Most European electricity trade occurs in day-ahead markets, cleared using EUPHEMIA. Due to the computational complexity of market clearing, power market models often simplify the process and bidding structures. As a result, advanced bidding formats – such as regular block bids, linked blocks, and loop blocks – are typically neglected, limiting the representation of storage traders' strategies. We propose a market coupling formulation incorporating advanced block orders to address this gap. Implemented in an agent-based power market model, we evaluate its impact across different scenarios. Our analysis reveals that including advanced bids increases computation time by roughly a factor of 10. However, this complexity increase can be mitigated by relaxing the fill-or-kill condition of block orders, which has minimal impact on dispatch outcomes.**

***Index Terms*—Market-Coupling Algorithm, Day-Ahead Market, Storage Systems, Block Bids, Linked Bids, Loop Bids**

I. Introduction

In 2014, European power exchanges integrated their day-ahead markets – the most liquid segment – by introducing the market clearing algorithm EUPHEMIA. This algorithm optimally allocates supply and demand while considering transmission constraints, thereby determining prices, traded volumes, and cross-border flows to maximize social welfare [1].

The day-ahead auction includes both simple hourly bids and advanced bidding formats. Simple bids, either piecewise linear or interpolated, result in a linear or quadratic optimization problem, respectively. Advanced orders – such as block, complex, and PUN orders – allow participants to incorporate technical (e.g. ramping limits) and economic constraints (e.g. minimum income) into their bids [2]. The fill-or-kill (FoK) constraint is a binary acceptance rule used in block orders and refers to a block order that must be either fully accepted or fully rejected – there is no partial acceptance. FoK of block orders introduce binary variables, resulting in mixed-integer problem. Merit and PUN orders further increase complexity due to strict consecutiveness constraints.

EUPHEMIA addresses this by decomposing market coupling into a master problem and three sub-problems. The master problem maximizes social welfare while initially relaxing FoK constraints for an upper welfare bound. If fractional block orders appear, EUPHEMIA iterates through a search tree to enforce binary acceptance. Additional sub-problems ensure rule-compliant prices, maintain Italy's uniform national price, and resolve volume indeterminacy [1].

EUPHEMIA computes daily auction results within minutes; publication by market operators follows 40 minutes after gate closure [2]. However, its computational complexity limits repeated executions, making it unsuitable for long-term analyses. E.g. a long term-analysis over 25 years requires nearly 10,000 daily market-coupling computions, making computational efficiency a key requirement. Consequently, alternative market-clearing methods are employed. Tanrisever et al. [3] classify these into direct, iterative, decomposition-based, heuristic, and dual-primal approaches. Market models like PowerACE [4] and AMIRIS [5] simplify market clearing by treating each hour independently, enhancing feasibility but omitting operational constraints. Wirtz et al. [2] address this by proposing a linear formulation incorporating block bids and minimum run times while relaxing the FoK condition. Conversely, Adams et al. [6] use an iterative method that maintains the FoK condition but increases complexity due to branching. Heuristic approaches employ cutting techniques [7], [8] or metaheuristics like tabu search and genetic algorithms [9]. Dual-primal models [10], [11] integrate primal variables (acceptance ratios) and dual variables (market-clearing prices) into a single MIP, embedding duality relationships directly.

Most advanced orders fail to capture the operational constraints of storage units, particularly the need for coordinated charging and discharging. This may force storage units to sell energy without prior acceptance of buy bids, risking infeasible dispatch. Introduced in 2018, loop block orders address this by linking buy and sell bids, ensuring storage units only discharge after accepted charge bids, thereby mitigating financial and operational risks [12].

This work formulates a market coupling formualtion incorporating block, linked, and loop bids. Building on [2], we identify FoK as a key complexity driver and propose two approaches: a mixed-integer programming (MIP) model enforcing FoK and a linearized relaxation. Both are integrated into an agent-based market simulation to assess their impact on market outcomes and computational complexity. The code is available at: https://gitlab.kit.edu/kit/iip/opensource/MarketClearingLoopBids.

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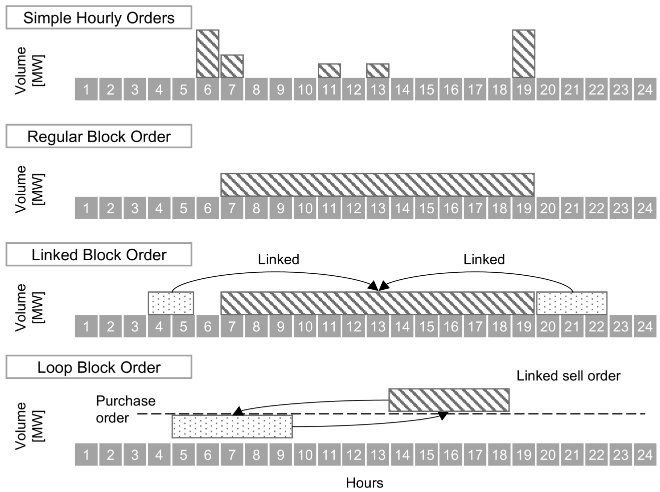

Fig. 1. Visualization of considered order types on a timeline [12]

II. Modelling of a Market Coupling Problem with Advanced Orders

A. Included Order Types

The subsequent market-clearing formulation includes bids as visualized in Fig. 1, accommodating both simple orders and advanced orders - specifically, regular, linked, and loop block orders. Simple bids represent the most basic bidding format, each consisting of a buy or sell order for a single trading period. They are specified by an hourly volume-price pair and can be partially accepted depending on the market clearing price. Regular block orders extend this concept by allowing traders to submit bids that span over multiple consecutive hours. Like simple bids, each block order has a specified volume and price, and can be formulated as either a buy or a sell bid. Block orders can be subject with or without FoK condition. The first case means the block acceptance ratios are binary leading to a MILP. The latter means that acceptance ratios can be in the interval $[0, 1]$.

Furthermore, block orders can be combined into more advanced bidding formats, such as linked and loop block orders:

- Linked block orders introduce a hierarchical structure in which a parent block order governs one or more child block orders. The acceptance ratio of the parent block sets an upper bound on the acceptance ratios of its child blocks.
- Loop block orders incorporate both buy and sell orders, pairing a demand block with a corresponding supply block. Unlike linked block orders, the acceptance ratios in loop block orders must be equal for the demand and supply blocks.

B. Problem Formulation

We present a European market coupling model based on [2], which we extend by adding loop blocks to represent storage. Our formulation furthermore allows the retrieval of market clearing prices. The objective function (1) maximizes social welfare across all bidding zones $m \in M$ and time steps $t \in T$. For each hourly offer $h \in H_{t,m}$, an acceptance ratio acc_h is introduced as a decision variable. Similarly, for each block offer $b \in B_{\tau,m}$, an acceptance variable acc_b is defined. The summation $\sum_{t \in T_\tau}$ represents the hours over which a block bid $\tau \in Z$ extends. Each demand offer contributes positively to overall welfare by increasing consumer surplus, while each supply offer reduces welfare by decreasing supplier surplus − both weighted by their respective bidding volume vol and price p.

$$\begin{aligned} \max \sum_{t \in T} \sum_{m \in M} \sum_{h \in H_{t,m}} -(vol_h * p_h) * acc_h \\ + \sum_{\tau \in Z} \sum_{m \in M} \sum_{t \in T_\tau} \sum_{b \in B_{\tau,m}} -(vol_b * p_b) * acc_b \end{aligned} \quad (1)$$

Constraint (2) ensures that each bidding zone is balanced including net exports $nex_{m,t}$, which are defined in (3). (4) guarantees a parity between supply and demand in the overall market, (5) restricts the flows between two bidding zones to the net transfer capacity $ntc_{i,j}$.

$$\begin{aligned} &\sum_{h \in H_{m,t}} vol_h acc_h + \sum_{b \in B_{\tau,m}} vol_b acc_b \\ &- nex_{m,t} = 0 \qquad \forall m \in M,\ t \in T \end{aligned} \quad (2)$$

$$\begin{aligned} nex_{m,t} = &\sum_{j \in M \setminus \{m\}} flow_{m,j,t} \\ &- flow_{j,m,t} \qquad \forall m \in M,\ t \in T \end{aligned} \quad (3)$$

$$\sum_{m \in M} nex_{m,t} = 0 \qquad \forall t \in T \quad (4)$$

$$0 \leq flow_{i,j,t} \leq ntc_{i,j} \qquad \forall i \neq j \in M,\ t \in T \quad (5)$$

(6) ensures that the acceptance ratio of simple orders lies within the interval [0,1]. Meanwhile, (7) introduces a relaxation of the FoK condition for block orders. This relaxation is crucial for preserving linearity in the problem, thus making it tractable within a linear optimization framework.

$$0 \leq acc_h \leq 1 \qquad \forall h \in H_{t,m} \quad (6)$$

$$0 \leq acc_b \leq 1 \qquad \forall b \in B_{\tau,m} \quad (7)$$

When block bids must fulfill FoK conditions, (8) enforces binary acceptance, transforming the clearing problem into a MILP.

$$acc_b \in \{0, 1\} \qquad \forall b \in B_{\tau,m} \quad (8)$$

(9) links the acceptance ratios of parent and child orders. (10) ensures that ask and buy orders in loop orders share the same acceptance ratio.

$$acc_b \leq acc_{\tilde{b}} \quad \forall b \in B_{\tau,m},\ \tilde{b} \in B_{\tilde{\tau},m},\ (b,\tilde{b}) \in LI \tag{9}$$

$$acc_b = acc_{\tilde{b}} \quad \forall b \in B_{\tau,m},\ \tilde{b} \in B_{\tilde{\tau},m},\ (b,\tilde{b}) \in LO \tag{10}$$

LI represents the set of linked blocks and LO represents the block tuples included in loop blocks.

C. Price Determination

The previously presented market clearing formulation does not explicitly model clearing prices. In general, prices can be derived by applying duality, which is commonly used to obtain hourly market clearing prices from the dual values of the bidding zone balance equations (2) [10]. However, in our model, each multi-hour block bid is represented by a single acceptance variable. Since the bid price, which could influence the market clearing price, is aggregated across all hours covered by the block, the corresponding elements of the dual problem collapse into a single-hour representation.

Algorithm 1 Post-processing step for price determination

foreach *bidding zone* z **do**
  Remove linked and looped block orders
  Identify price-setting order on the supply side
  Identify price-setting order on the demand side
end
foreach *pair of bidding zones* (i, j) **do**
  if $flow_{i,j} < ntc_{i,j}$ **then**
    Merge bidding zones i and j into the same pricing zone
  else
    Keep bidding zones i and j as separate pricing zones
  end
end
foreach *pricing zone* p **do**
  Determine market clearing price based on last accepted bids
end
return Market clearing prices for all zones

As a result, the derived dual prices fail to capture the actual hourly marginal values, making duality-based price retrieval infeasible in this framework. One way to address this problem is to introduce an acceptance variable for each hour of the block bid rather than using a single acceptance variable for the entire block. This approach avoids aggregating bid prices and allows the correct retrieval of shadow prices for each hour. However, this alternative formulation significantly increases the number of constraints and decision variables. Moreover, if the FoK condition of block orders is modeled, this leads to a MILP formulation, which inherently prevents the application of duality due to the discrete nature of the optimization problem.
Hence, we opt for a different approach and derive the prices in a post-processing step similar to [13] and as shown in Algorithm 1. The price-defining order on both the supply and demand side is identified for each bidding zone. To determine uniform pricing zones, the flow $flow_{i,j}$ between two bidding zones is compared to the corresponding net transfer capacity $ntc_{i,j}$. If $flow_{i,j} < ntc_{i,j}$ indicating the capacity is not fully utilized, the bidding zones share the same market clearing price. Otherwise, they form separate pricing zones. This procedure is repeated pairwise for all bidding zones.
However, deriving prices based on the last accepted bids may lead to inaccuracies when bid families are involved since blocks can compensate the losses of each other. A visualization of such edge cases, that are accepted despite their individual position in the merit order, is provided in Fig. 2. To mitigate this, linked and looped block orders are excluded during the post-processing phase of price retrieval. Although this approach can be solved computationally easily, it causes potential imprecisions in prices and potentially paradoxically accepted blocks (PABs). The latter refers to blocks that are accepted even though they potentially cause unintentional losses for the provider as they violate the pricing rule (e.g., a supply block order that is accepted despite an offer price higher than the market clearing price). These occur in case a block order as part of a loop or linked family is price-setting but excluded due to our algorithm. The minor price deviations then cause PABs.

Additionally, if regular block orders exist, partially accepted blocks emerge as soon as the intersection of supply and demand occurs at the price level of a regular block order. Whereas an MILP formulation avoids this case, a relaxed LP does not.

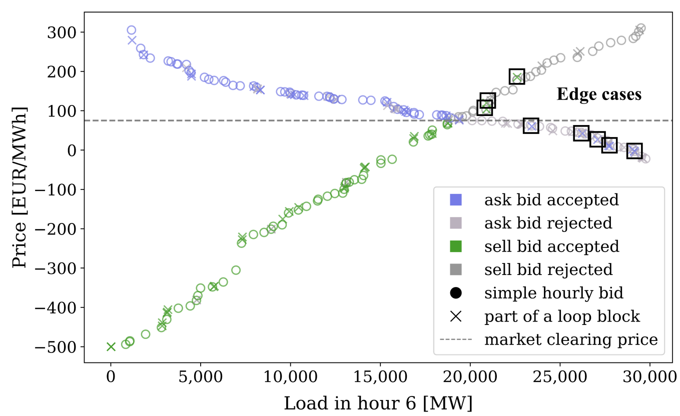

Fig. 2. Exemplary merit order highlighting edge cases

III. Analysis of Problem Formulation

To evaluate the relaxed formulation, which results in a LP problem, and the FoK formulation, which leads to a MILP market coupling formulation, a simplified test environment was implemented within PowerACE. The setup consists of two large bidding zones and one smaller zone, with randomly generated bids. The setup includes two large bidding zones and a smaller one, with randomly generated bids. Prices follow a left-skewed logarithmic transformation with an expected value of 80 €/MWh, reflecting Germany's 2024 day-ahead market

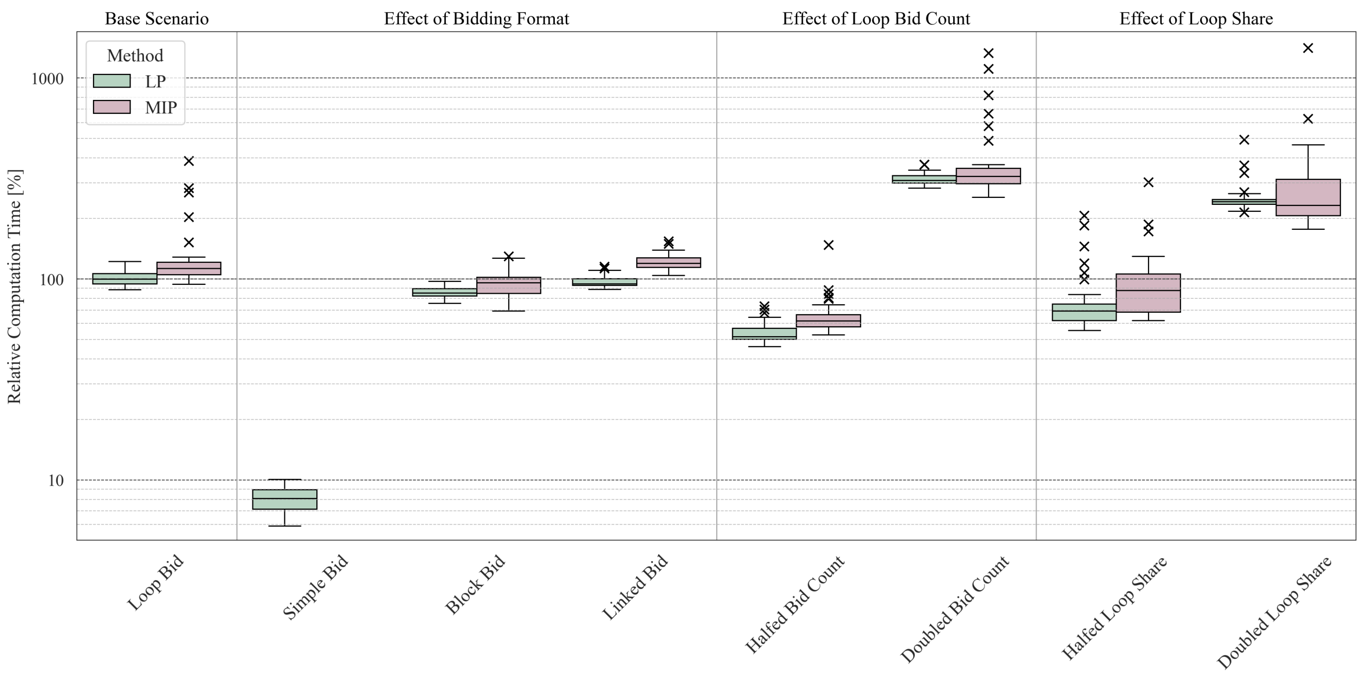

Fig. 3. Run time comparison between linear formulation and MIP across multiple scenarios

price. All bids are within the price ranges of the day-ahead market of $p \in [-500; 4000]$. Simple hourly orders and block order start times are drawn from a bimodal distribution peaking at 7 a.m. and 4 p.m. Block order durations follow a uniform distribution, ensuring they do not extend into the following day. Regular block orders, modeled as supply bids with 50 % lower prices on average, simulate large fossil power plants' inflexibilities [2], [6]. Loop and linked orders combine a buy and sell block with a 40 % price spread to secure provider revenue. Each simulation covers a full year (365 days), with optimization solved using Gurobi 11.0.1 on an Intel Core i5-1235U (2.50 GHz). The solver was set up to find an optimal solution with a MIP gap of zero.

A. *Evaluation of Run Time*

To compare computational performance, various scenarios with different bid types and configurations were simulated (see Appendix Table I for an overview). Scenario 1 serves as the baseline, featuring 12,500 loop bids and 37,500 simple bids. In Scenario 2, 3, and 4, the loop bids from the baseline are replaced by simple bids, block bids, and loop bids, respectively. Scenarios 5 and 6 return to the baseline loop bid setting but alter the total number of bids, halving it in Scenario 5 and doubling it in Scenario 6. Finally, Scenarios 7 and 8 examine the impact of the proportion of loop bids among the total number of bids, halving this share in Scenario 7 relative to the baseline and doubling it in Scenario 8. Furthermore, each scenario that contains block orders was modeled both as an LP (no FoK condition) and as an MIP (with FoK condition). The results of the simulation are included in Fig. 3. To improve comparability in the plotting, the run time of the different scenarios is plotted relative to the run time of the median of the baseline LP loop bids scenario.

Advanced bidding types significantly increase computational complexity. Simple bids allow for efficient parallelization, whereas time-coupled block orders impose the highest computational burden. Hence, the time-coupled optimization increases running times by a factor of 8.4 (comparing scenario 1 and 2). Linked and loop bids add constraints beyond standard block orders, with loop bids being the most demanding due to stricter equality constraints. Increasing the market coupling problem size, represented by bid count, directly affects computation time. Doubling the bids significantly extends run time, reflecting an exponential growth in complexity, while halving them reduces computational effort accordingly. Higher loop bid shares increase computation time and variability, as seen in the boxplot, while lower shares reduce computational effort. This highlights the strong dependency between loop bid proportion and run time fluctuations. The introduction of the FoK constraint transforms the market coupling problem into a MIP, significantly increasing computational complexity. Enforcing strict binary acceptance values extends computation time, since fractional blocks require branching. In a market with 100,000 overall bids, for instance, the simulation of one year extends from 37.9 minutes in the LP formulation to 48.4 minutes in the MIP. Considering a typical simulation horizon of 30 years for power market models such as PowerACE, this run time increase of approximately 21.7% is significant. Furthermore, the number of outliers with exceptionally high run times increases substantially in the MIP compared to the LP formulation, highlighting reduced robustness in handling the integrated bidding structure. This effect arises because scenarios with many fractional bids force the algorithm to iterate multiple times to find a feasible solution, leading to prolonged solving times. A growing integration of smart bidding strategies in power markets could lead to an increase of such cases.

B. Market Clearing Result

To evaluate the proposed problem formulation, the market clearing result is examined in more detail, as depicted in Fig. 4. A higher number of block orders in the overall market leads to a higher number of partially accepted blocks in the relaxed LP formulation on average. This is expected as the probability increases that the intersection of the step-wise cumulative demand and supply curves falls on a partially accepted regular block bid. Additionally, linkages and loops between regular blocks cause PABs as they induce the edge cases described above. However, putting the total numbers of PABs with our approach into perspective, we can conclude that even with 12,500 linked or loop blocks in the market in total, PABs do not exceed a number of 10 at maximum which is 0.08 % of the total number of block bids. Assuming a growing number of bidding zones this increases but is still manageable. We further compare the relaxed LP formulation to its MIP benchmark in a welfare and a dispatch analysis. Due to its structure, a relaxation always represents the upper bound of its corresponding MIP in a maximization case, and so does our problem formulation. The anaylsis shows that the welfare difference between both optimization approaches does not exceed 0.0001 % in all cases and the number of bids with differing acceptance ratios, respectively dispatch, does not exceed 0.05 %, respectively 37 bids per day. Thus, the relaxed LP provides reasonably accurate results with a significant run time avantage in market scenarios with a large number of overall bids. Additionally, the analysis shows that linked and especially loop blocks smooth out daily price spreads in our case studies. This result is crucial because it suggests that linked and loop blocks can reduce volatility on exchanges. Lastly, experiments support the risk mitigation potential of loop blocks for individual traders. Reproducing markets of scenario 1 with independent ask and sell blocks shows a significant number of bids with differing acceptance ratios.

IV. DISCUSSION AND OUTLOOK

This work proposes a computationally efficient approach to incorporate advanced order types such as regular, linked, and loop block orders in a power market clearing algorithm. This approach allows the inclusion of operational constraints of storage units in long-term power market models such as PowerACE. The resulting market clearing formulation is analyzed in exemplary market scenarios and benchmarked in run time and dispatch.

Observations indicate that complexity mainly arises from the time-coupling of hourly optimization problems, which hinders multithreading. The inclusion of FoK orders has only a minor impact on runtime, as partially accepted block bids remain rare. However, this may change as market participants, such as storage operators, increasingly utilize advanced bidding strategies like loop orders. In loop orders, sell bids can be placed at higher prices, increasing both the likelihood of partial acceptance and the probability of influencing price formation. In MIP scenarios, all blocks were modeled with FoK, which may be unnecessary for flexible units like storage, reducing complexity. Linkages and loops between block orders require accounting for interdependencies in the clearing process, adding complexity - especially with loop blocks, which impose tighter constraints.

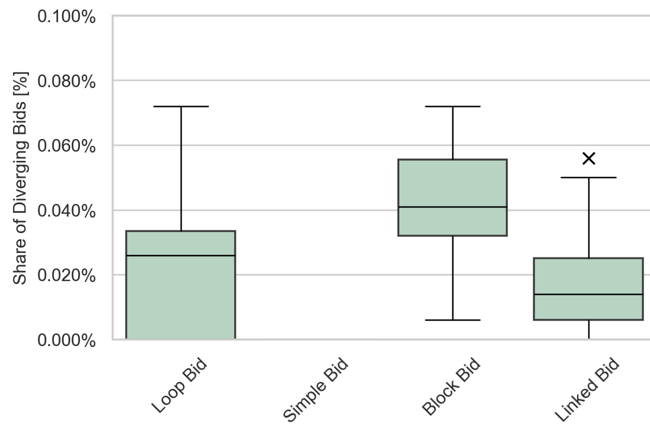

Fig. 4. Comparison of LP and MILP Market Clearing: Number of Differing Bids

The interaction of looped and linked block orders creates edge cases that can distort the price determination process, affecting less than 0.1 % of block bids in our simulations. EUPHEMIA addresses these cases through a dedicated price-determination sub-problem, albeit at the cost of significantly increased runtime. However, since PABs are rare, we do not consider a separate sub-problem necessary for power market model simulations.

In conclusion, advanced bids increase the complexity of power market clearing - a process that will continue as power markets reduce their Market Time Unit and as the number of storage traders increases. Using loop blocks poses a solution for risk mitigation strategies of storage traders as price spreads included in such advanced offers are secured. Thus, a positive revenue is guaranteed if the order is executed. If the order is out of the market vice versa, no losses occur.

Looking ahead, while advanced orders like loop bids mitigate price risks, dispatch decisions remain with the bidder, limiting dispatch flexibility from the perspective of the market operator. This is particularly a crucial factor for large storages. More advanced bids (such as exclusive orders) address this by shifting responsibility to the clearing algorithm, enabling better optimization. Hence, integrating such bidding formats into market coupling problems could further improve the integration of flexibilities into market models.

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Appendix

Table I: Overview of Analyzed Scenarios

Scenario	**Total Bids**	**Share of Hourly Demand**	**Share of Hourly Supply**	**Share of Block Supply**	**Share of Linked Blocks**	**Share of Loop Blocks**
1: Loop Bid	50,000	37.5%	37.5%	0%	0%	25%
2: Simple Bid	50,000	50%	50%	0%	0%	0%
3: Block Bid	50,000	37.5%	37.5%	25%	0%	0%
4: Linked Bid	50,000	37.5%	37.5%	0%	25%	0%
5: Halfed Bid Count	25,000	37.5%	37.5%	0%	0%	25%
6: Doubled Bid Count	100,000	37.5%	37.5%	0%	0%	25%
7: Halfed Loop Share	50,000	43.75%	43.75%	0%	0%	12.5%
8: Doubled Loop Share	50,000	25%	25%	0%	0%	50%